Math 151 Problem Set #2 for Chapter 3. C. S. Davis

Name ____

1. For the polynomial $y = p(x) = -x^4 + 4x^3 + 3x^2 - 18x = (x - 3)^2 (x) (x + 2)$ do the following.
   a. give the general shape for a polynomial of this degree and this leading coefficient;
   b. give the maximum number of possible humps;
   c. choose a window that will show all of the important features or the polynomial and graph the function on your graphing calculator (and, of course, transfer the window and graph to your paper labeling the axes with numbers and letters.)
   d. find the intervals where the function is positive or negative;
   e. find the intervals where the function is increasing and decreasing.

2. Use synthetic division to divide polynomials, evaluate polynomials and test for roots.

Consider the polynomial $f(x) = 5x^3 - 7x^2 - 28x + 12$. Use synthetic division to do each of the following.

a. divide and simplify $\dfrac{f(x)}{x-4} = \dfrac{5x^3 - 7x^2 - 28x + 12}{x-4}$;

b. write f(x) in the form $f(x) = D(x)\,Q(x) + R(x)$

c. find the value of f(4);

d. determine whether 3 is a root of f;

e. determine whether -2 is a root of f;

f. determine whether 5 is a root of f.

g. check parts b-e with the graphing calculator.

3. Give the rational, irrational and complex roots and their multiplicities for the polynomial $p(x) = 6(x-3)^4(x+7)x^2(x^2-10x+34)$ given here in factored form.

4. Construct the polynomial whose roots are $x = 3 \pm i, 2, 0$ and which passes through the point $(1,40)$. Expand the polynomial.

5. Give the possible rational roots for the polynomial $p(x) = 2x^4 - 9x^3 + 46x^2 - 89x + 34$ with integer coefficients.

6. Find all rational, irrational and complex roots and their multiplicities of the polynomial $p(x) = 2x^4 - 9x^3 + 46x^2 - 89x + 34$ given here with integer coefficients. Use the table feature of the calculator and synthetic division to test whether the possible rational roots are actual roots. Check your answers with the graphing feature of the calculator.

7. For the polynomial $f(x) = 49.0\,x^4 + 21.0\,x^3 - 122.0\,x^2 + 60.0\,x - 8.0$ given here with real coefficients, do the following:

   a. give the overall shape and maximum possible number of humps.
   b choose a window that will show all of the important features or the polynomial and graph the function (and, of course, transfer the window and graph to your paper labeling the axes with numbers and letters.)
   c. use the trace and zoom-in feature to approximate real roots to 4 significant figures;
   d. use poly (on the TI-86) to find the real and complex roots and their multiplicities;
   e. describe the connection that the roots and multiplicities have with the graph.

8. For the rational function $f(x) = \dfrac{x^3 + 3x^2 - 2x + 5}{x^2 - x - 3}$

a. Find the equation of and draw the graph of the vertical asymptotes (dotted lines). (Use the quadratic formula on this one as the denominator does not factor easily.)

b. Find the equation of and draw the graph of the non-vertical asymptotes (dotted lines).

c. Graph the function (solid curve).

d. Check the graph using the graphing calculator.

9. For the rational function $f(x) = \dfrac{2x^3 - 20x^2 + 15x}{x^3 + x^2 - 5x + 3}$ draw the graph using the grap

a. Use the window and/or zoom features to show all important features.

b. Use the trace and zoom-in features to approximate the roots to several significant figures.

c. Approximate the equations of the horizontal and vertical asymptotes.

10. Perform the arithmetic of these two complex numbers and graph them and their conjugates.

a. Simplify $(2 + 5i) + (3 - i)$

b. Simplify $(2 + 5i) - (3 - i)$

c. Simplify $(2 + 5i) \times (3 - i)$

d. Simplify $(2 + 5i) / (3 - i)$

e. Graph $(2 + 5i)$ and $(3 - i)$ and both of their complex conjugates. Label the axes with names and numbers.

11. Use mathematical induction to prove that the following formula is true for all natural numbers n.

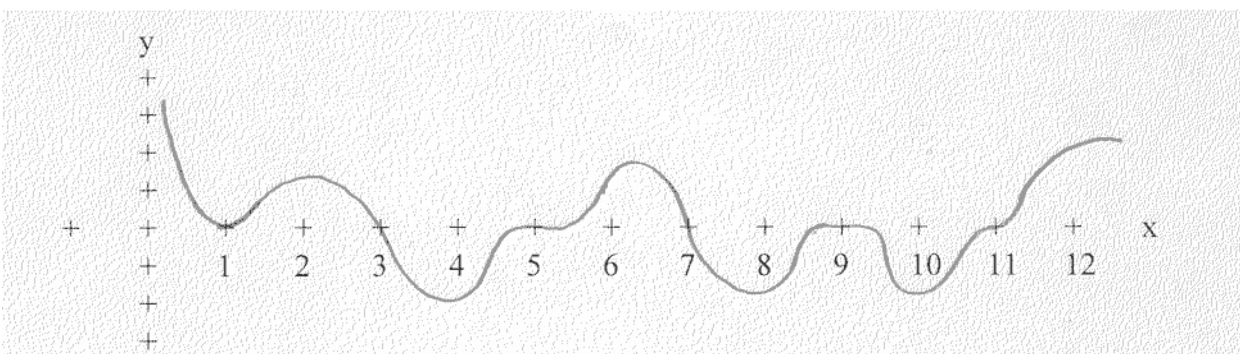
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

12. Expand $(4n - 1)^5$ using the binomial theorem. (Using Pascal's triangle is okay on this one.)

13. a. In the binomial expansion of $(2 + x)^9$ give the terms containing x^0, x^1, x^2, x^3 . (Use the binomial coefficients from the formula on this one.)

    b. Check your answer using x = .01 to see how close the first four terms come to the correct answer.

14. Given the graph of a polynomial below, estimate the roots and their multiplicity.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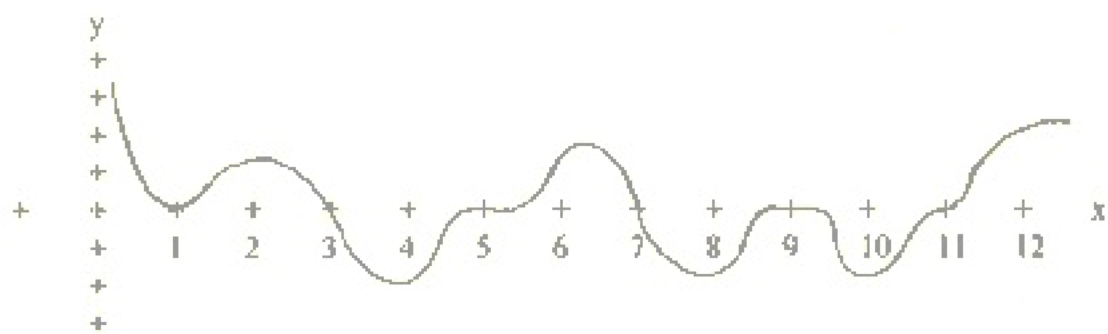
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