M 191 Lect #1 1-21-09

While I'm taking roll, please find the slope of the line passing through (2,1) and (4,6). Then write some forms of the equation of the line.

| x | y |
|---|---|
| 4 | 6 |
| 2 | 1 |

$$m = \frac{6-1}{4-2} = \frac{5}{2} = 2.5 = 2\tfrac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{5}{2}(x-2)$$

$$y = \frac{5}{2}x - \frac{5}{\cancel{2}}\cdot\cancel{2} + 1$$

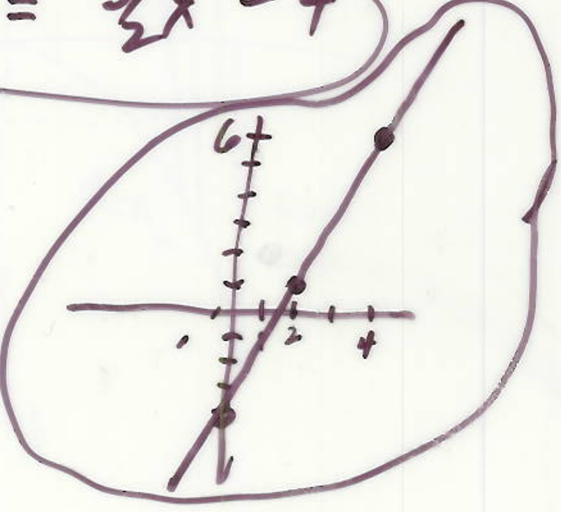
$$y = \frac{5}{2}x - 4$$

$$2(y-1) = 5(x-2)$$

$$2y - 2 = 5x - 10$$

$$-5x + 2y = -8$$

$$5x - 2y = 8$$

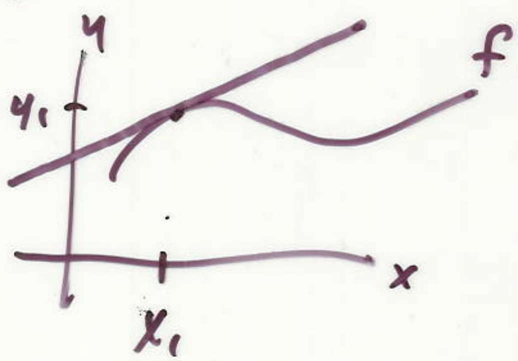



$$m = \frac{5}{2}$$

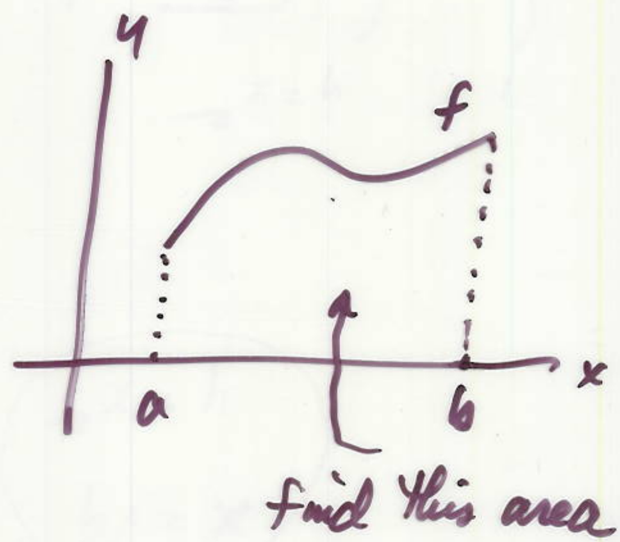
$$(x_1, y_1) = (2,1)$$

I want first to tell you what the two major themes of Calculus are.

① Find the equation of the <u>tangent</u> line to a curve at a given point




② Find the area under this curve

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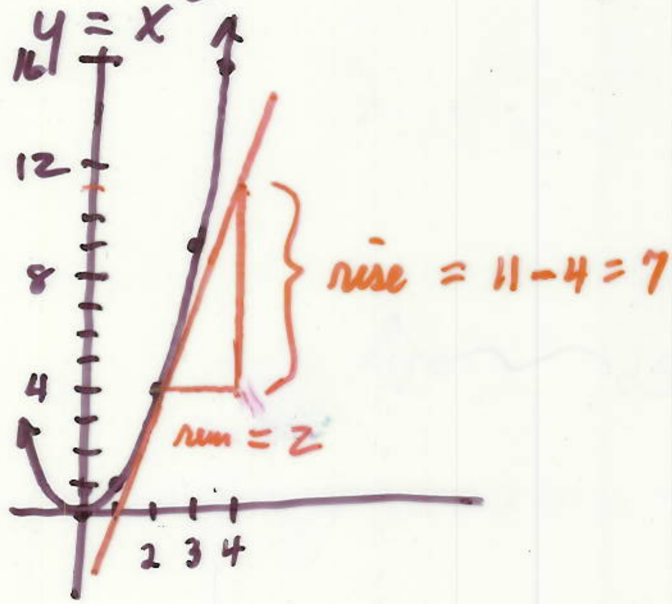
Let's find the slope and the equation of the tan line to the curve $y = f(x) = x^2$ at $x = 2$ three ways

① graphically
② numerically
③ algebraically

① tan's slope graphically at $x=2$




| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

$$m_{tan} \doteq \frac{7}{2}$$

Eq of tan line is about

$$y - y_1 = m_{tan}(x - x_1)$$

$$\boxed{y - 4 = \tfrac{7}{2}(x-2)}$$

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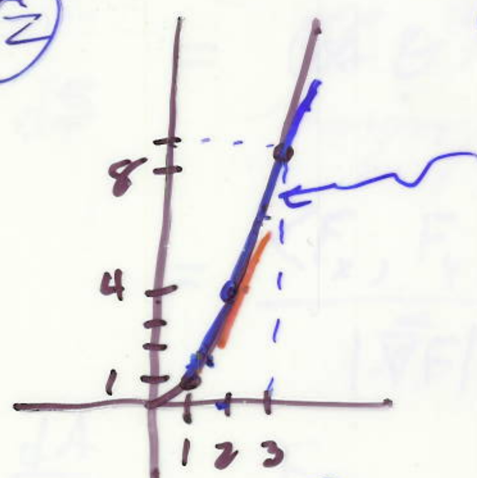
② Numerically




$$m_{sec1} = \frac{4-9}{2-3} = \frac{-5}{-1} = 5$$

$$m_{sec2} = \frac{1-4}{1-2} = \frac{-3}{-1} = 3$$

$$m_{tan} \overset{\approx}{\doteq} \text{ave} = \frac{m_{sec1} + m_{sec2}}{2} = \frac{5+3}{2} = 4$$

eq of tan line is about $y - 4 = 4(x-2)$

③ <u>Algebraically</u>

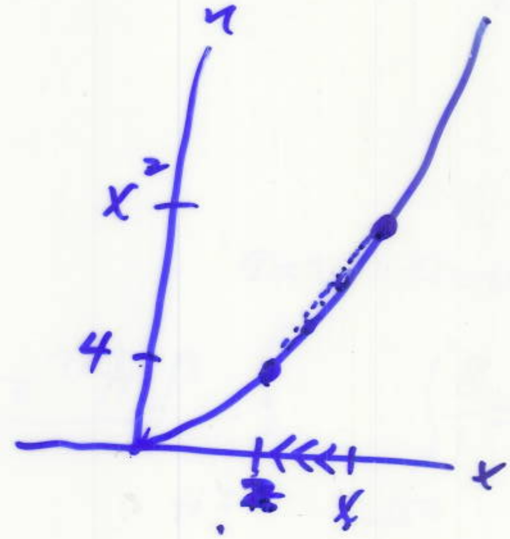
$f(x) = x^2$

| x | y |
|---|---|
| x | $x^2$ |
| 2 | 4 |

$$m_{sec} = \frac{x^2-4}{x-2}$$

$$= \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \quad \text{as long as } x \neq 2$$

$$m_{sec} = x+2$$




$$m_{tan} \doteq m_{sec}$$

$$m_{tan} = \lim_{x\to 2} m_{sec}$$

$$= \lim_{x\to 2} x+2$$

$$= 4$$

So the real eq of the tan line is

$$y-4 = 4(x-2)$$

Now we move to the study of limits in a more general setting than just $\lim m_{sec} = m_{tan}$

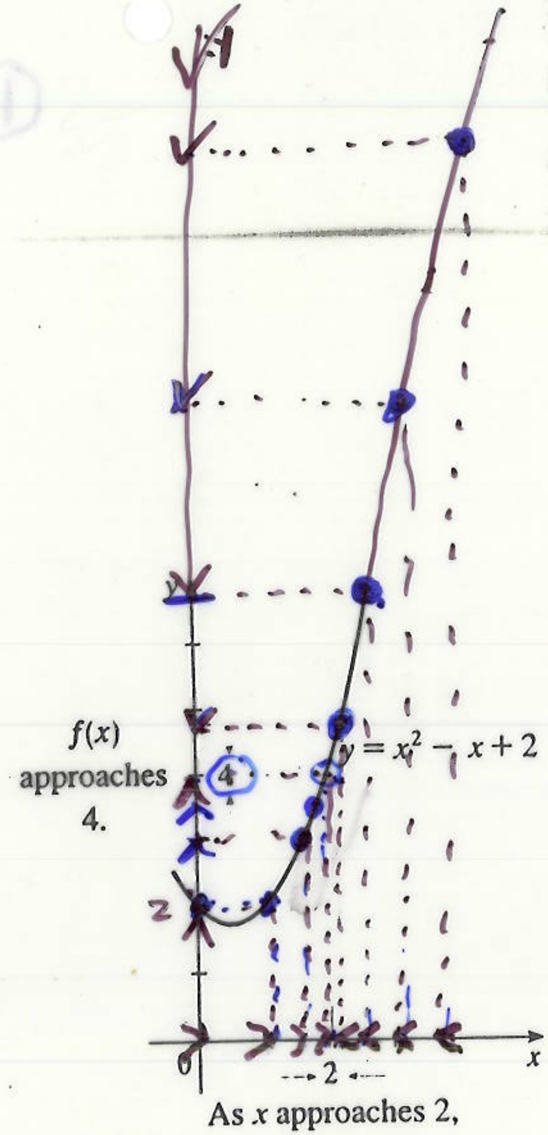



FIGURE 1

| $x$ | $f(x)$ | $x$ | $f(x)$ |
|---|---|---|---|
| 1.0 | 2.000000 | 3.0 | 8.000000 |
| 1.5 | 2.750000 | 2.5 | 5.750000 |
| 1.8 | 3.440000 | 2.2 | 4.640000 |
| 1.9 | 3.710000 | 2.1 | 4.310000 |
| 1.95 | 3.852500 | 2.05 | 4.152500 |
| 1.99 | 3.970100 | 2.01 | 4.030100 |
| 1.995 | 3.985025 | 2.005 | 4.015025 |
| 1.999 | 3.997001 | 2.001 | 4.003001 |

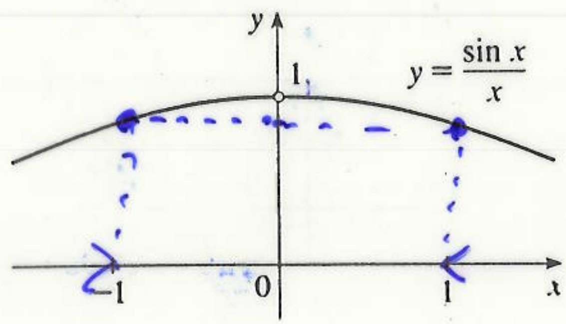
$$\lim_{x \to 2} f(x) = 4$$

~~P70~~ P66

p6

(2)

| $x$ | $\frac{\sin x}{x}$ |
|---|---|
| ±1.0 | 0.84147098 |
| ±0.5 | 0.95885108 |
| ±0.4 | 0.97354586 |
| ±0.3 | 0.98506736 |
| ±0.2 | 0.99334665 |
| ±0.1 | 0.99833417 |
| ±0.05 | 0.99958339 |
| ±0.01 | 0.99998333 |
| ±0.005 | 0.99999583 |
| ±0.001 | 0.99999983 |

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$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(3)

| $x < 1$ | $f(x)$ |
|---|---|
| 0.5 | 0.666667 |
| 0.9 | 0.526316 |
| 0.99 | 0.502513 |
| 0.999 | 0.500250 |
| 0.9999 | 0.500025 |

| $x > 1$ | $f(x)$ |
|---|---|
| 1.5 | 0.400000 |
| 1.1 | 0.476190 |
| 1.01 | 0.497512 |
| 1.001 | 0.499750 |
| 1.0001 | 0.499975 |

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$\lim_{x \to 1} f(x) = .5$

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4. For the function $f$ whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.

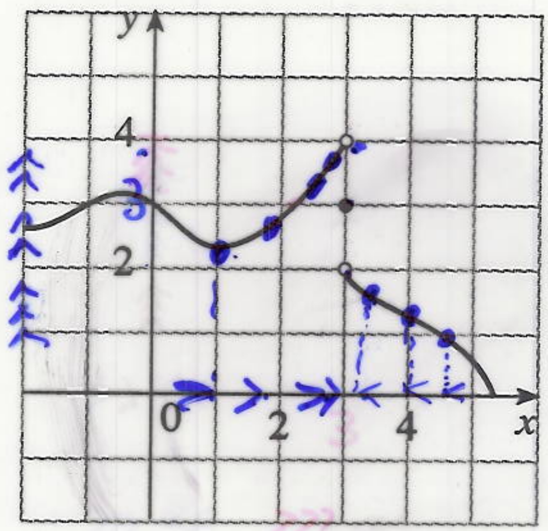
T (a) $\lim_{x \to 0} f(x) = 3$

D (b) $\lim_{x \to 3^-} f(x) = 4$

T (c) $\lim_{x \to 3^+} f(x) = 2$

L (d) $\lim_{x \to 3} f(x)$ · Does Not Exist - DNE

I (e) $f(3) = 3$

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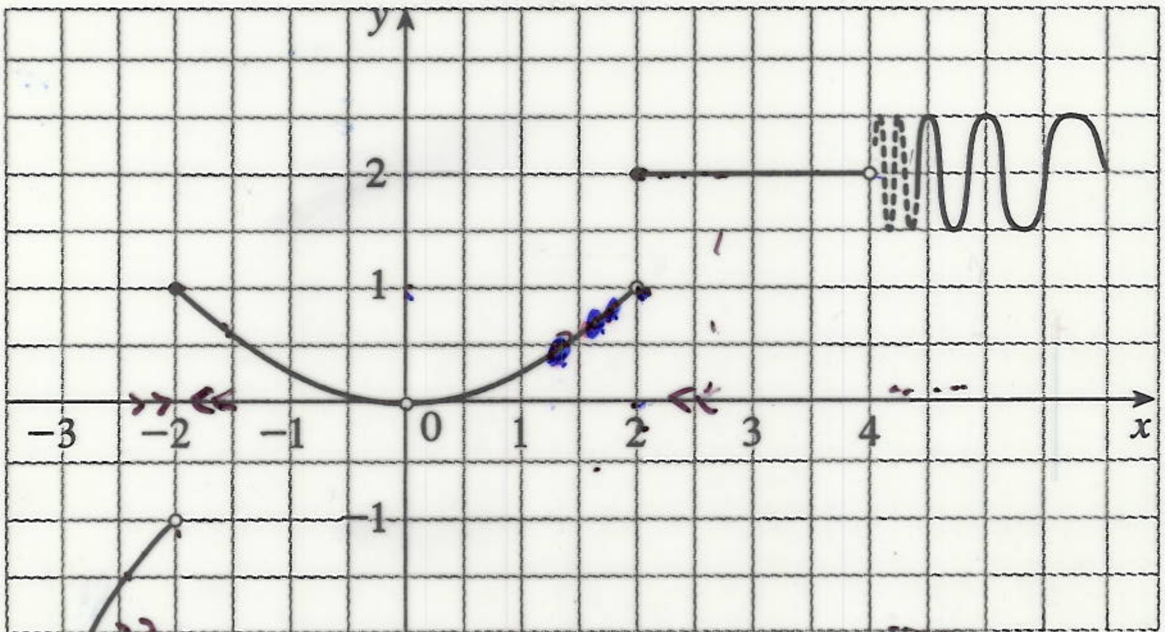
**6.** For the function $g$ whose graph is given, state the value of the given quantity, if it exists. If it does not exist, explain why.

M (a) $\lim_{x\to -2^-} g(x)$ $=-1$  A (b) $\lim_{x\to -2^+} g(x)$ $=1$  G (c) $\lim_{x\to -2} g(x)$ DNE since $1\neq -1$




M (d) $g(-2)$ $=1$  A (e) $\lim_{x\to 2^-} g(x)$ $=1$  R (f) $\lim_{x\to 2^+} g(x)$ $=2$

R (g) $\lim_{x\to 2} g(x)$ DNE  T (h) $g(2)=2$  J (i) $\lim_{x\to 4^+} g(x)$ DNE

C (j) $\lim_{x\to 4^-} g(x)$ $=2$  J (k) $g(0)$ DNE  T (l) $\lim_{x\to 0} g(x)=0$

$2+\frac{1}{2}\sin\left(\frac{1}{x-4}\right)$