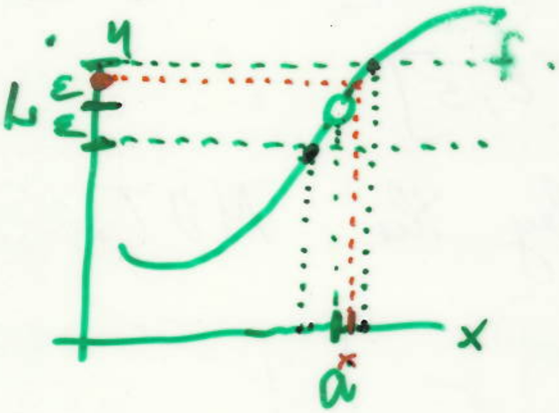
M 191 Lect #3 1-31-11



δ ε

Let f be a fcn defined in an interval containing $x=a$ except maybe not at $x=a$.
The limit of f as x approaches a is L if

① for every $\varepsilon > 0$
② there exists a $\delta > 0$
③ such that whenever $0 < |x-a| < \delta$
④ then $|f(x) - L| < \varepsilon$

We do our proofs in two phases

Discovery
1
4
3
2

Verification
1
2
3
4

Use the definition of limit (ε-δ process) to prove that

$$\lim_{\underbrace{x \to 2}_{a}} \underbrace{7x-3}_{f(x)} = \underbrace{11}_{L}$$

<u>Discovery Phase</u>

① Let $\varepsilon > 0$

④ Require $|f(x) - L| < \varepsilon$

Req $|7x - 3 - 11| < \varepsilon$

Req $|7x - 14| < \varepsilon$

Req $7|x-2| < \varepsilon$

③ Req $|x-2| < \frac{\varepsilon}{7} \overset{\text{set}}{=} \delta$

② Let $\delta = \frac{\varepsilon}{7} > 0$

<u>Verification Phase</u>

① Let $\varepsilon > 0$

② Let $\delta = \frac{\varepsilon}{7} > 0$

③ Let x be chosen to satisfy $0 < |x-2| < \delta = \frac{\varepsilon}{7}$

④ then $|f(x) - L|$

$= |7x - 3 - 11|$

$= |7x - 14|$

$= 7|x-2|$

$< 7 \cdot \delta$

$= 7 \cdot \frac{\varepsilon}{7}$

$= \varepsilon$

Use the limit laws to evaluate this limit

$$\lim_{x\to 2} \frac{7x^2(5x+3)^6}{\sqrt{6x-1}}$$ by $\boxed{5}$ lim of quot

$$= \frac{\lim_{x\to 2} 7x^2(5x+3)^6}{\lim_{x\to 2} \sqrt{6x-1}}$$ by $\boxed{4}$ lim of prod

$$= \frac{\lim_{x\to 2} 7x^2 \cdot \lim_{x\to 2} (5x+3)^6}{\sqrt{\lim_{x\to 2} (6x-1)}}$$ by $\boxed{11}$ lim of root

$$= \frac{7 \cdot \lim_{x\to 2} x^2 \cdot \left(\lim_{x\to 2} (5x+3)\right)^6}{\sqrt{\lim_{x\to 2} 6x - \lim_{x\to 2} 1}}$$ by $\boxed{3}$ by $\boxed{6}$ by $\boxed{2}$

$$= \frac{7 \cdot 2^2 \cdot \left(\lim_{x\to 2} 5x + \lim_{x\to 2} 3\right)^6}{\sqrt{6 \lim_{x\to 2} x - 1}}$$ by $\boxed{9}$ by $\boxed{1}$ by $\boxed{3}$ by $\boxed{7}$

$$= \frac{7 \cdot 2^2 \cdot \left(5 \lim_{x\to 2} x + 3\right)^6}{\sqrt{6 \cdot 2 - 1}}$$ by $\boxed{7}$

$$= \frac{7 \cdot 2^2 \cdot (5 \cdot 2 + 3)^6}{\sqrt{6 \cdot 2 - 1}} = 4826809.??$$

There are roughly 7 behaviors for curves of functions

	graph	$y=$	Cont @ $x=0$
1 smooth		x^2	✓
2 corners		PW $\lvert x\rvert$	✓
3 cusps		$-x^{\frac{2}{3}}$	✓
4 vert tan		$x^{\frac{1}{3}}$	✓
5 jumps disc		PW $\begin{cases} x & x\geq 0 \\ 3-x & x<0 \end{cases}$	✗
6 holes + Dots removable disc		$\begin{cases} \frac{x^2-2x}{x} = \frac{x(x-2)}{x} & x\neq 0 \\ 5 & x=0 \end{cases}$	✗
7 vert asymptotes infinite disc		$y=\frac{1}{x^2}$	✗
			✓