M 191 Lect #4 2-2-11

Consider $f(x) = 9x + 2$. Let $a = 5$

Guess $\lim_{x \to a} f(x) = \lim_{x \to 5} (9x+2) = 47$

Use the def of limit (ε-δ process) to prove the above

<u>Discovery Phase</u>

① Let $\varepsilon > 0$

④ Require $|f(x) - L| < \varepsilon$

Req $|9x+2-47| < \varepsilon$

Req $|9x-45| < \varepsilon$

Req $9|x-5| < \varepsilon$

$|x-5| < \frac{\varepsilon}{9} = \delta$

③ Let $\delta = \varepsilon/9 > 0$

②

<u>Verify Phase</u>

① Let $\varepsilon > 0$

② Let $\delta = \frac{\varepsilon}{9} > 0$

③ Let x satisfy

$0 < |x-5| < \delta = \frac{\varepsilon}{9}$

④ Then $|f(x) - L|$

$= |9x+2-47|$

$= |9x-45|$

$= 9|x-5|$

$< 9\delta$

$= 9 \cdot \varepsilon/9$

$= \varepsilon$

Let f be a function defined near $x=a$.

f is continuous at $x=a$ if

$$\lim_{x \to a} f(x) = f(a)$$

① $\lim_{x \to a^-} f(x)$ exists $= L_1$

② $\lim_{x \to a^+} f(x)$ exists $= L_2$

③ $f(a)$ is defined $= N$

④ $L_1 = L_2 = N$

$$f(x) = \begin{cases} x-4 & x<2 \\ x^2 & x=2 \\ 2x-1 & x>2 \end{cases}$$

Is f cont @ $x=2$?

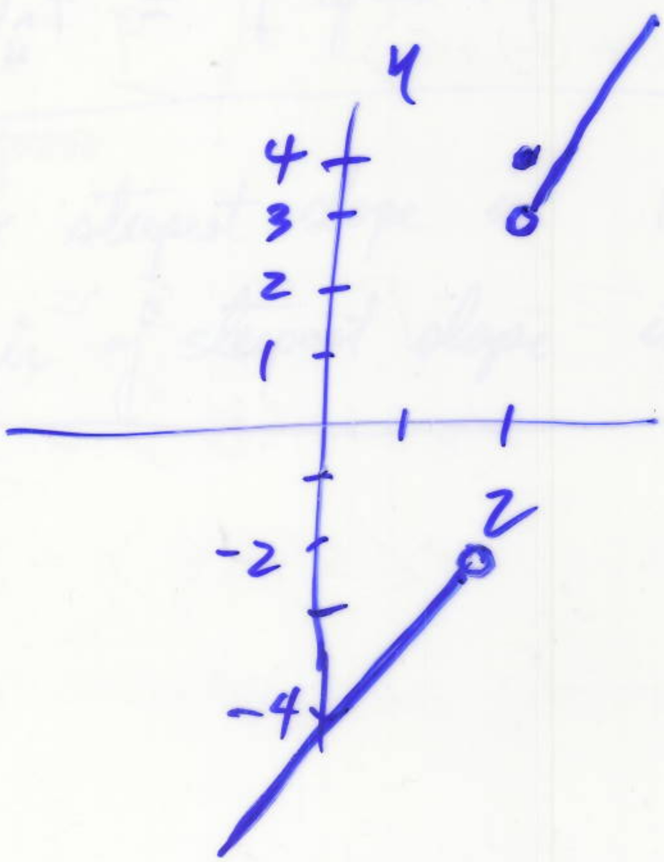
① $\lim\limits_{x \to 2^-} (x-4) = 2-4 = -2 = L_1$

② $\lim\limits_{x \to 2^+} (2x-1) = 2 \cdot 2 - 1 = 3 = L_2$

③ $f(2) = 2^2 = 4 = N$

④ $-2 \neq 3 \neq 4$

Hence f is not cont @ $x=2$



jump disc @ $x=2$

two holes

p 4

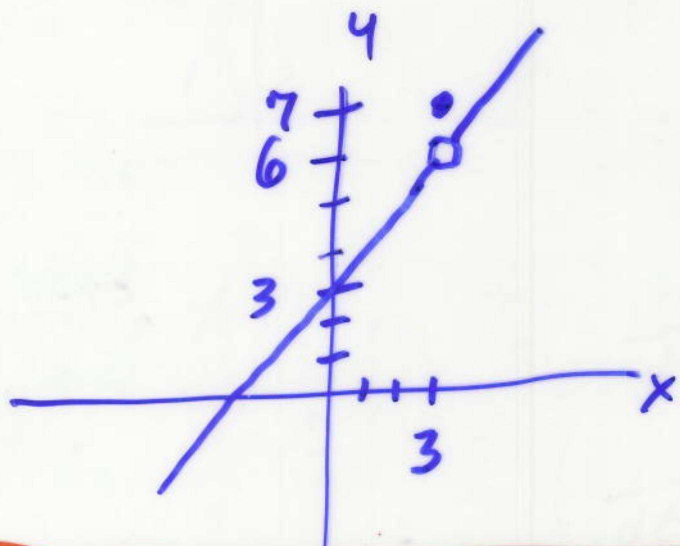
$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 7 & x = 3 \end{cases}$$

Is f cont @ $x=3$

① ② $\lim_{x \to 3} \dfrac{x^2-9}{x-3} = \lim_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 3+3=6$

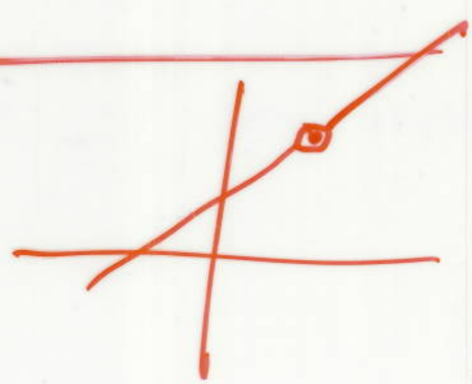
③ $f(3) = 7$

④ $6 \neq 7$ Hence f is not cont @ $x=3$



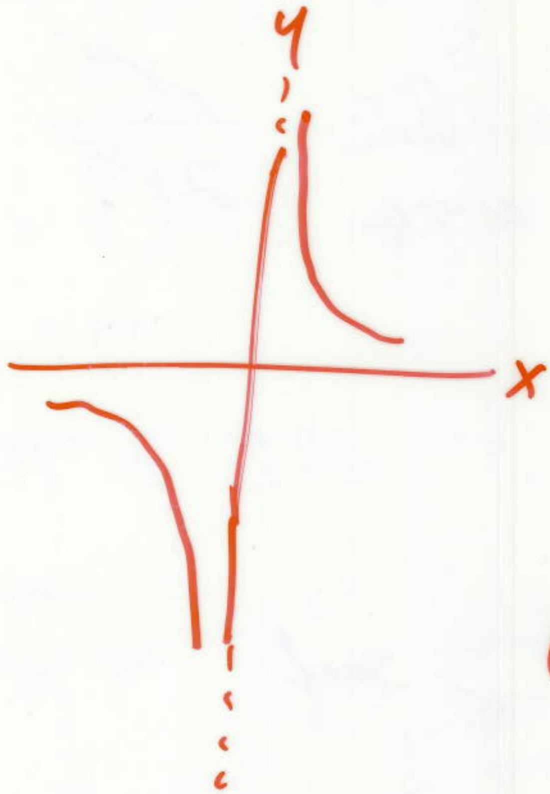
removable disc

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x \neq 3 \\ 6 & x=3 \end{cases}$$



f is cont @ $x=3$

$f(x) = \frac{1}{x^3}$ Is f cont @ $x=0$?



f is not cont @ $x=0$

① $\lim_{x \to 0^+} \frac{1}{x^3} = +\infty$

② $\lim_{x \to 0^-} \frac{1}{x^3} = -\infty$

③ $f(0)$ is undef

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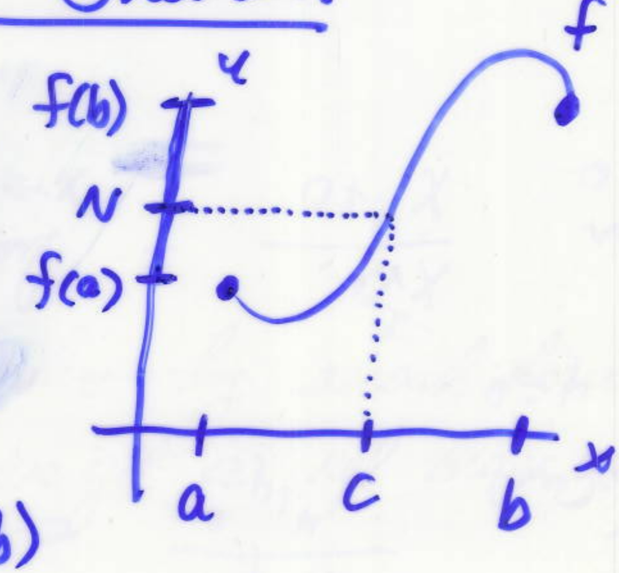
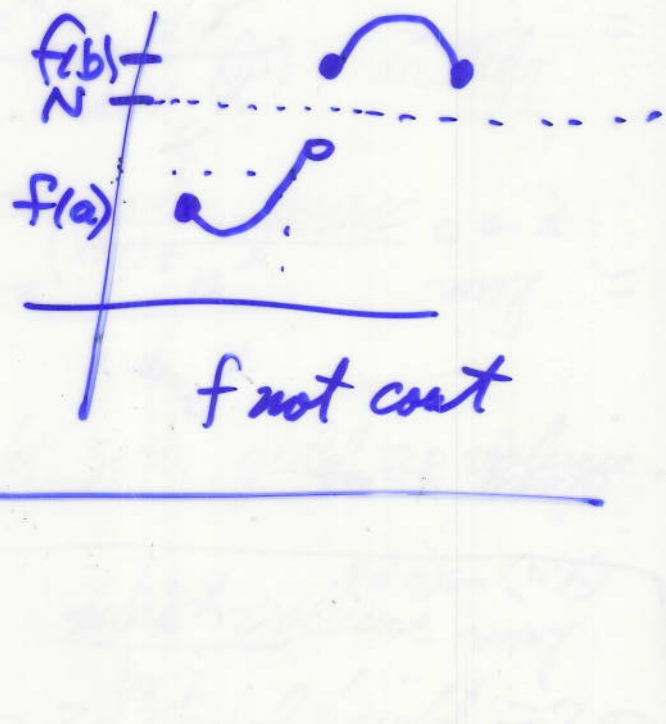
f is cont on an interval (a,b) or $[a,b]$ if it is cont at every point $a < x < b$ or on $a \le x \le b$

Intermediate Value Theorem

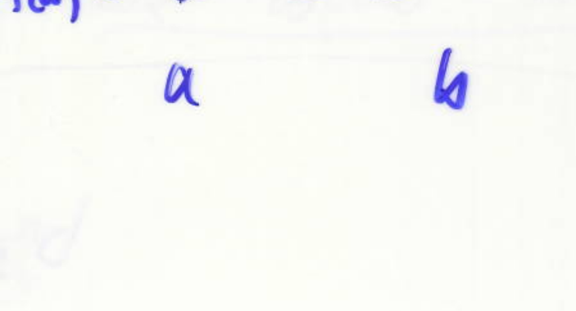
Let f be a cont fcn on $[a, b]$.



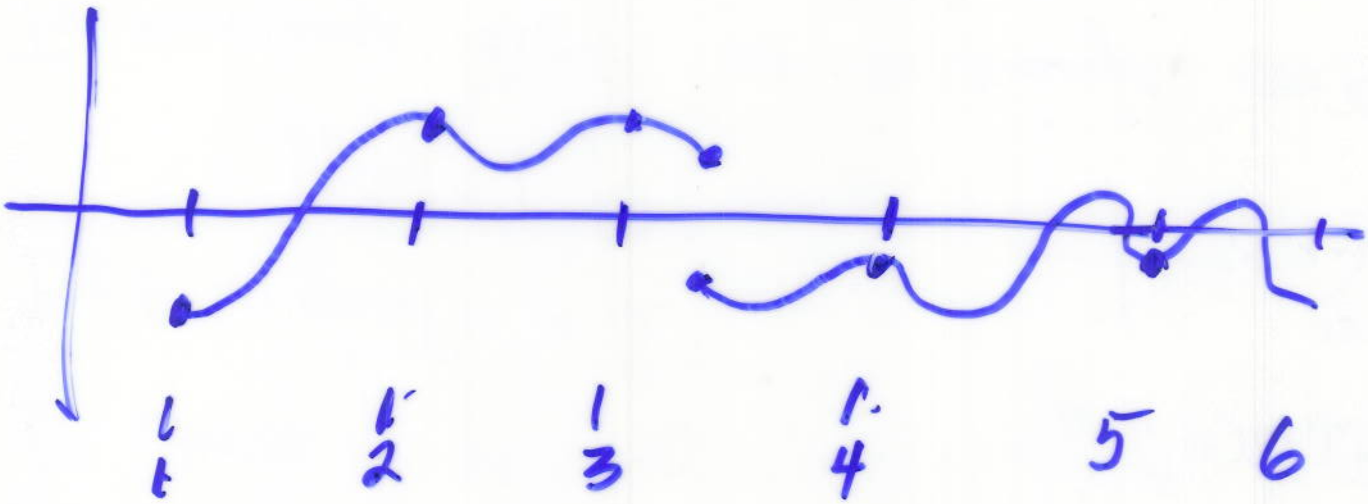
For any number N between $f(a) \leq N \leq f(b)$ there is an $x = c$ such that $f(c) = N$



We use the IVT most of the time this way



Fun Picture



On $[1,2]$, IVT guar f has a zero

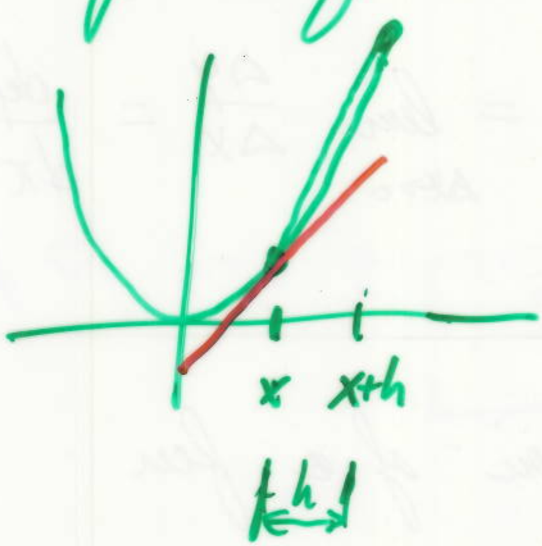
$[2,3]$ " D.N.A.

$[2,4]$ " DNA f not cont on $[2,4]$

$[4,5]$ " DNA $N=0$ not betw $f(a)$ & $f(b)$

Recalling lecture #1 we want to find the slope of the secant line m_{sec} and the slope of the tan line m_{tan} for any x value of the fcn.

$f(x) = x^2$



x	y
x+h	$(x+h)^2$
x	x^2

h

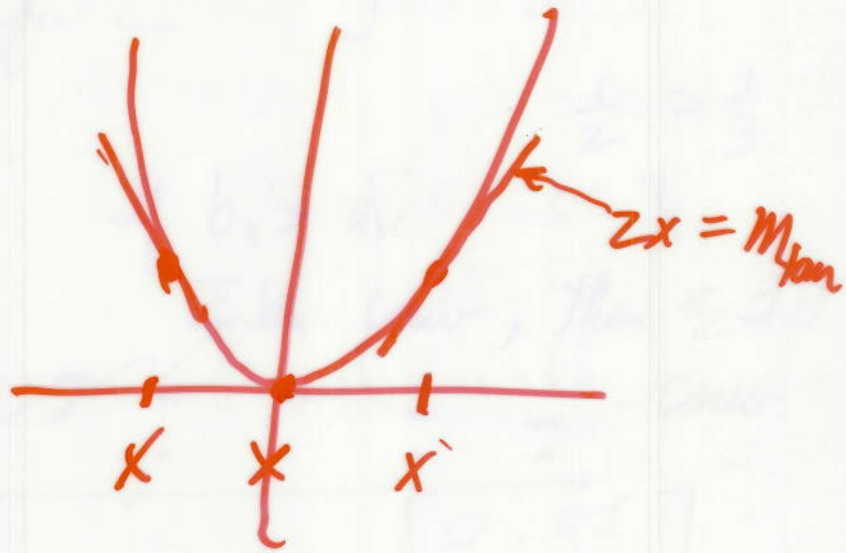
$$m_{sec} = \frac{(x+h)^2 - x^2}{x+h-x} = \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$m_{sec} = \frac{2xh+h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$m_{sec} = 2x+h$$

$$m_{tan} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} 2x+h$$

$$m_{tan} = 2x$$



Generalizing to any fcn $y=f(x)$

x	y
$x+h$	$f(x+h)$
x	$f(x)$

h

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^2 + 3x - 5$

find m_{tan}

x	y
$x+h$	$(x+h)^2 + 3(x+h) - 5$
x	$x^2 + 3x - 5$

h

$$m_{tan} = \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - 5 - (x^2 + 3x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{-5} \cancel{-x^2} \cancel{-3x} \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h + 3 = 2x + 3$$

In calculus we give the name of m_{tan} to be the <u>derivative</u> of f

$$f'(x) = m_{tan} = 2x+3$$

$$y' = 2x+3$$

$$\frac{dy}{dx} = 2x+3$$