M 191 Lect #7 2-14-11

The Product Rule

$$f(x) = F(x) \cdot S(x) = (4x^2+5)(9x^4+3x^2)$$

$$f'(x) = F(x) \cdot \text{der } S(x) + S(x) \cdot \text{der } F(x)$$

$$= F \cdot S' + S \cdot F'$$

$$= (4x^2+5) \cdot (9 \cdot 4x^3 \cdot 1 + 3 \cdot 2x^1 \cdot 1)$$

$$+ (9x^4+3x^2) \cdot (4 \cdot 2x^1 \cdot 1 + 0)$$

$$= (4x^2+5)(36x^3+6x) + (9x^4+3x^2)(8x)$$

$$= 144x^5 + 24x^3 + 180x^3 + 30x + 72x^5 + 24x^3$$

$$= 216x^5 + 228x^3 + 30x$$

$$y = (3x^1 + 5)(x^2 - 1)$$

$$\frac{dy}{dx} = y' = (3x+5)(2x \cdot 1) + (x^2 - 1)(3 \cdot 1x^0 \cdot 1)$$

$$= 6x^2 + 10x + 3x^2 - 3$$

$$= 9x^2 + 10x - 3$$

Quotient Rule

$$f(x) = \frac{N(x)}{D(x)} = \frac{D \cdot \text{der}\, N - N \cdot \text{der}\, D}{D^2}$$

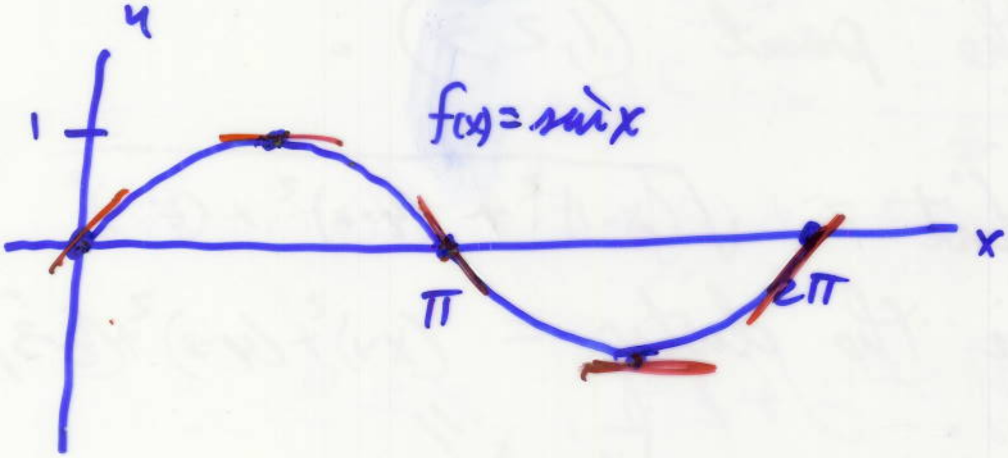
$$f(x) = \frac{4x+9}{6x-3}$$

$$f'(x) = \frac{(6x-3)(4 \cdot 1) - (4x+9)(6 \cdot 1)}{(6x-3)^2}$$

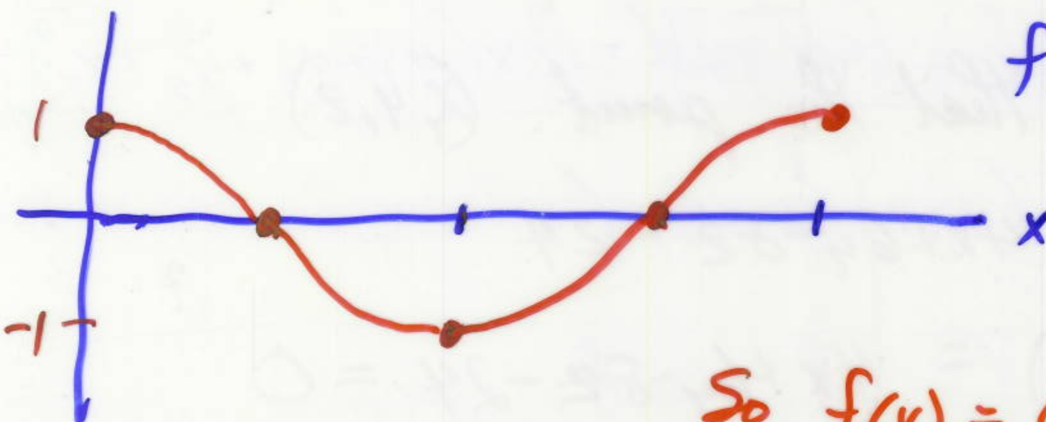
$$= \frac{2x - 12 - (24x + 54)}{(6x-3)^2} = \frac{-66}{(6x-3)^2} = \frac{-\overset{22}{\cancel{66}}}{\underset{3}{\cancel{9}}(2x-1)^2}$$

We want to find derivatives of trig functions.

$f(x) = y = \sin x$



$f'(x) = \frac{d}{dx} \sin x$



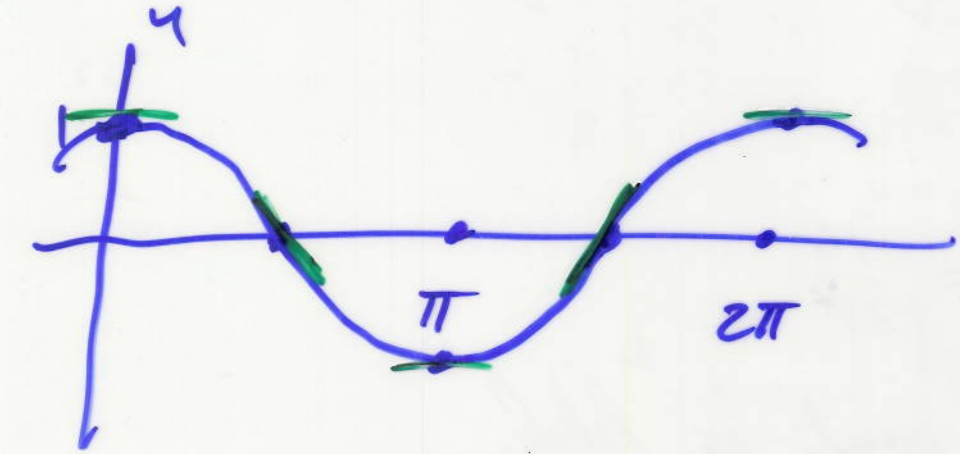
So $f(x) = \sin x$

$f'(x) = \cos x \cdot 1$

der of $\sin x = \cos x \cdot 1$

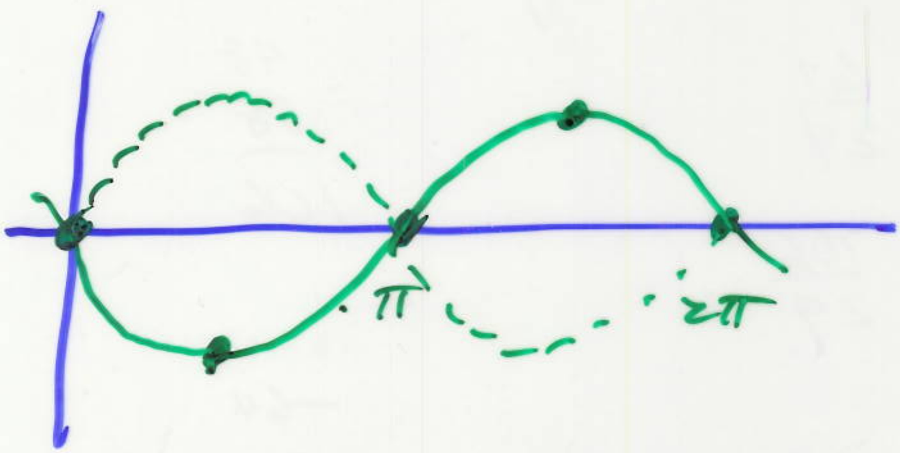
$\frac{d}{dx} \sin x = \cos x$

$f(x) = \cos x$



$$f'(x) = \frac{d}{dx} \cos x$$

$$= -\sin x$$



$$\frac{d}{dx} \cos x = -\sin x \cdot 1$$

Table of elementary fcn

$f(x)$	$f'(x)$
x^n	$nx^{n-1}\cdot 1$
$\sin x$	$\cos x \cdot 1$
$\cos x$	$-\sin x \cdot 1$
$\tan x$	$\sec^2 x \cdot 1$
$\cot x$	$-\csc^2 x \cdot 1$
$\sec x$	$\sec x \cdot \tan x \cdot 1$
$\csc x$	$-\csc x \cdot \cot x \cdot 1$

Practice

$$f(x) = \sin x \cdot (x^2 + 3x)$$

$$= \sin x \cdot (2x \cdot 1 + 3 \cdot 1) + (x^2 + 3x) \cos x \cdot 1$$

$$= (2x+3) \sin x + (x^2+3x) \cos x$$

$$\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} = \frac{\cos x (\cos x \cdot 1) - (\sin x)(-\sin x \cdot 1)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{D \cdot \text{der } N - N \cdot \text{der } D}{D^2}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x \cdot 1$$

$$\frac{d}{dx} \left(x^8 \cdot \tan x \right) = x^8 \cdot \sec^2 x \cdot 1 + \tan x \cdot 8x^7 \cdot 1$$

$$\frac{d}{dx} \frac{\cot x}{x^3 + 5x} = \frac{(x^3+5x) \cdot (-\csc^2 x \cdot 1) - \cot x (3x^2 + 5 \cdot 1)}{(x^3+5x)^2}$$

$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x}$$

$$= \frac{\cancel{\cos x}(0)^{\to 0} - 1(-\sin x \cdot 1)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \cdot \tan x \cdot 1$$

Composition of Functions

x	x
$g(x)$	x^2+3x
$f(g(x))$	$2(x^2+3x)$
$e(f(g(x)))$	$\left(2(x^2+3x)\right)^3+5\left(2(x^2+3x)\right)$

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x

$g(x) = x^2+3x$

$f(x) = 2 \cdot x$

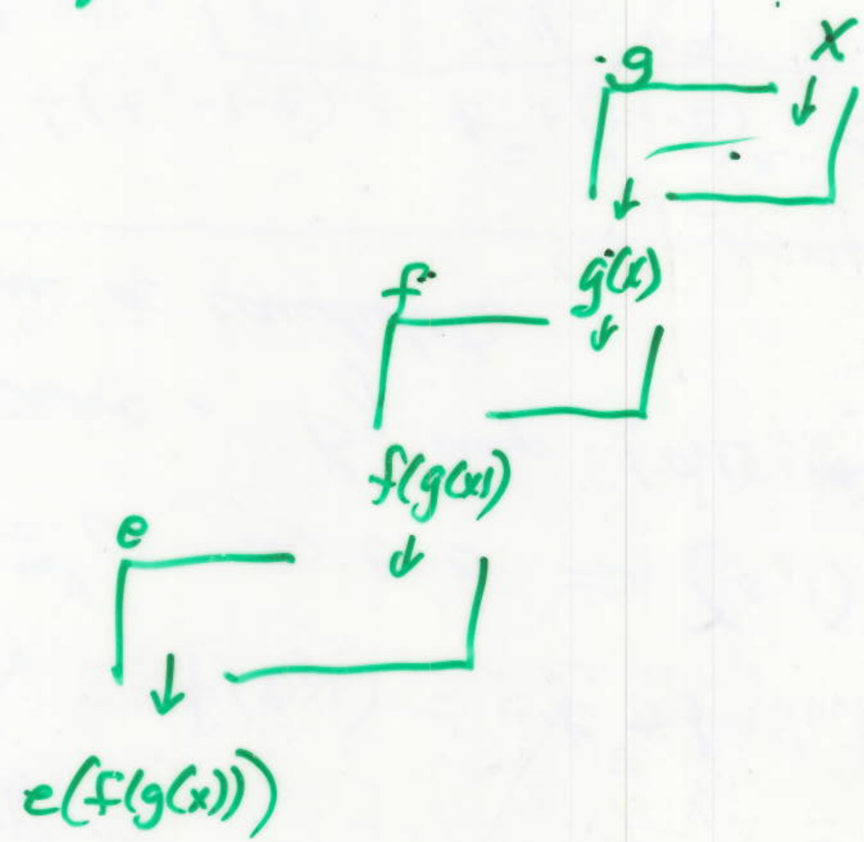
$e(x) = x^3+5x$

$g(x) = x^2+3x$

$f(g(x)) = 2(x^2+3x)$

$e(f(g(x))) = \left(2(x^2+3x)\right)^3+5\left(2(x^2+3x)\right)$

Machine Diagram



x

$$g(x) = x^2 + 3x$$

$$f(x) = \sin(x)$$

$$f(g(x)) = \sin(x^2 + 3x)$$

$$\frac{d}{dx} f(g(x)) = \cos(x^2 + 3x) \cdot (2x + 3) \cdot 1$$

$$= f'(g(x)) \cdot g'(x) \cdot 1$$

$$\frac{d}{dx}\sqrt{7x^4+5x^2}$$

$$\frac{d}{dx}\left(7x^4+5x^2\right)^{1/2} = \frac{1}{2}\left(7x^4+5x^2\right)^{-1/2}\cdot\left(7\cdot 4x^3+5\cdot 2x\right)$$

$$= \frac{28x^3+10x}{2\sqrt{7x^4+5x^2}}$$

$$\frac{d}{dx}\frac{2x+5}{(x^3+8x)^7} = \cancel{\frac{(x^3+8x)^7(2)-(2x+5)7(x^3+8x)^6(3x^2+8)}{(x^3+8x)^{14}}}$$

$$x$$

$$u = g(x) \qquad y = f(g(x)) = (f\circ g)(x)$$

$$y = f(u) \qquad \frac{dy}{dx} = f'(g(x))\cdot g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \cancel{\frac{dy}{du}\cdot\frac{du}{dx}}$$