M 191 Lect #8 2-16-11

Let's see the d form of the chain rule

$y = f(\underbrace{g(x)}_{u}) \longrightarrow u = g(x)$

$y = f(u) \qquad u = g(x)$

$\frac{dy}{du} = f'(u) \qquad \frac{du}{dx} = g'(x)$

$\frac{dy}{dx} = \boxed{\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \cdot 1}$

$\frac{dy}{dx} = f'(u) \cdot g'(x) \cdot 1$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

$y = f(m) \quad m = g(p) \quad p = h(q) \quad q = i(x)$ p^2

$y = m^2+1 \quad m = 4p-5 \quad p = q^4+q^2 \quad q = \sin(x)$

$$y = f(g(h(i(x))))$$

$$\frac{dy}{dx} = \frac{dy}{dm}\,\frac{dm}{dp}\,\frac{dp}{dq}\,\frac{dq}{dx}$$

$$\frac{dy}{dx} = (2m)(4)(4q^3+2q)(\cos x)\cdot 1$$

Implicit Differentiation

Mostly our equations are explicitly defined like this

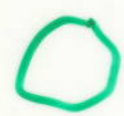
$$y = f(x) \qquad y = x^3 + 4x + 7$$

Sometimes like with circles we like our relationship this way

$$x^2 + y^2 = 16$$

If we solve for y

we would get



$$y^2 = 16 - x^2$$

$$y = \pm\sqrt{16 - x^2}$$

Find y' in this expression
(Use implicit diff.)

$$x^2 + y^2 = 1$$

Diff w/r x

$$2x \cdot \cancel{\frac{dx}{dx}}^{\,1} + 2y \cdot \frac{dy}{dx} = \cancel{\frac{d\,1}{dx}}^{\,0}$$

Solve for $\frac{dy}{dx}$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$m_{tan} = y' = \frac{dy}{dx} = -\frac{x}{y}$$

Implicitly diff to find $\frac{dy}{dx} = y'$

$$x^3 \cdot y^5 + \cos x + \tan y = 8$$

$$x^3 \cdot 5y^4 \cdot \frac{dy}{dx} + y^5 \cdot 3x^2 \cdot \cancel{\frac{dx}{dx}}' - \sin x \cdot \cancel{\frac{dx}{dx}}'$$

$$+ \sec^2 y \cdot \frac{dy}{dx} = \frac{d8}{dx}$$

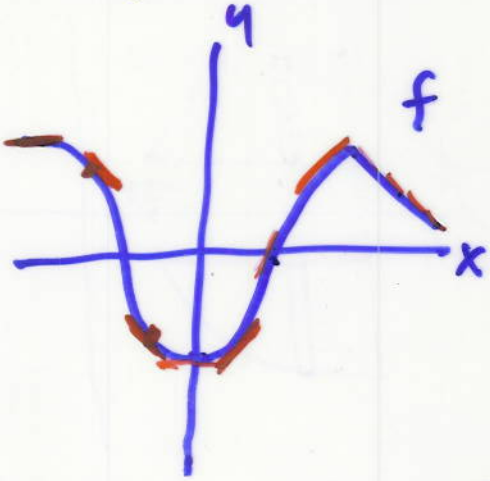
$$5x^3y^4 \cdot y' + 3x^2y^5 - \sin x + \sec^2 y \cdot y' = 0$$

$$\left(5x^3y^4 + \sec^2 y\right) y' = \sin x - 3x^2y^5$$

$$m_{tan} = \frac{dy}{dx} = y' = \frac{\sin x - 3x^2y^5}{5x^3y^4 + \sec^2 y}$$

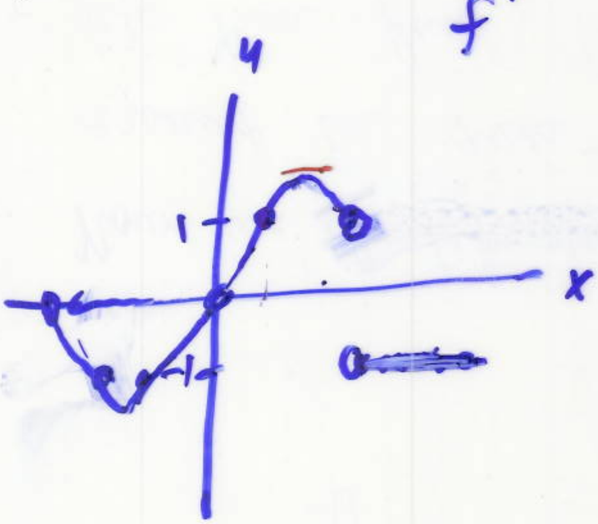
We want to find higher order derivatives

Graphically | Numerically | Algebraically

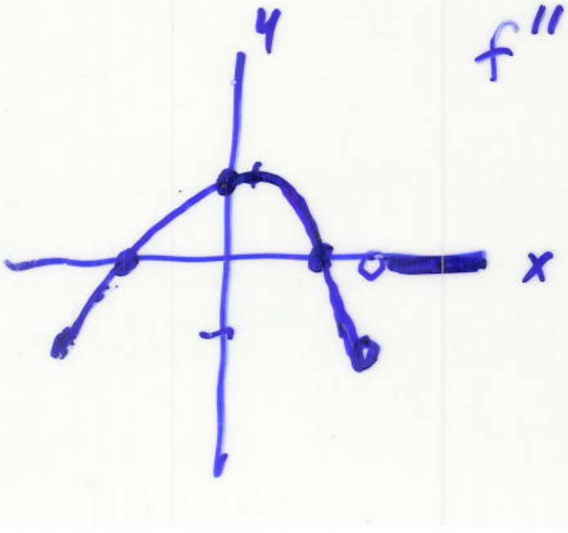


For the original curve $y=f(x)$

$f(x)$ is the height formula



$f'(x)$ is a slope formula



$f''(x)$ is a _____ formula

$f'''(x)$ is a ________ formula

Numerically

x	$f(x)$	$f'(x)$	$f''(x)$
2	40	$\frac{50-40}{4-2} = 5$	$\frac{10-5}{4-2} = \frac{5}{2}$
4	50	$\frac{80-40}{6-2} = 10$	$\frac{10-5}{6-2} = \frac{5}{4}$
→ 6	80	$\frac{90-50}{8-4} = 10$	$\frac{-5-10}{8-4} = -\frac{15}{4}$
8	90	$\frac{60-80}{10-6} = -5$	$\frac{-15-10}{10-6} = \frac{-25}{4}$
10	60	$\frac{60-90}{10-8} = -15$	$\frac{-15-(-5)}{10-8} = -5$

Algebraically

$$f(x) = x^5 + 7x^3$$

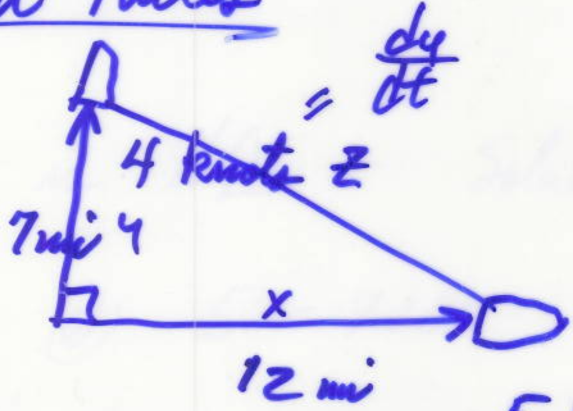
$$f'(x) = 5x^4 \cdot 1 + 21x^2 \cdot 1$$

$$f''(x) = 20x^3 \cdot 1 + 42x \cdot 1$$

$$f'''(x) = 60x^2 \cdot 1 + 42 \cdot 1$$

$$f''''(x) = 120x \cdot 1$$

Related Rates



$5 \text{ knots} = \frac{dx}{dt}$

$$x^2 + y^2 = z^2$$

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$$\boxed{\begin{gathered} x \cdot \frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt} \\ x^2 + y^2 = z^2 \end{gathered}}$$

When $x = 12$, $y = 7$, $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 4$ Find $\frac{dz}{dt}$

$$\boxed{\begin{gathered} 12 \cdot 5 + 7 \cdot 4 = z\frac{dz}{dt} \\ 12^2 + 7^2 = z^2 \end{gathered}} \rightarrow 60 + 28 = 88 = \sqrt{193}\,\frac{dz}{dt}$$

$z^2 = 144 + 49 = 193$, $z = \sqrt{193}$

$\frac{dz}{dt} = \frac{88}{\sqrt{193}} \approx 6.3$ knots