M 191 Lect #12 3-2-11

Closed Interval Extrema Th

Let f be a fcn that is defined & cont on [a, b]. Then the Abs extrema will be found at one or more of these places

① where deriv is zero

$f'(x) = 0$ x is called a Type I critical value

② where deriv is undefined

$f'(x)$ is undf x is called a Type II critical value

③ at an endpoint of the interval

$x = a$ or $x = b$.

Ex Find Abs Extrema for

$y=f(x) = x^3-6x^2+9x+2$ on the interval $[2,5]$

① $f'(x) = 3x^2-12x+9 \overset{\text{set}}{=} 0$

$$3(x^2-4x+3) = 0$$

$$x^2-4x+3=0$$

$$(x-1)(x-3) = 0$$

$x=1, 3$ are CV's for f

Aha, $x=1$ is not in specified dom $[2,5]$

So Now we only consider $x=3$ CV.

② polynomials are diff. everywhere so no TII CV's

③ EP's are $x=2,5$.

④ Make Table

	x	$y = f(x) = x^3 - 6x^2 + 9x + 2$	
EP	2	$4 = 2^3 - 6\cdot 2^2 + 9\cdot 2 + 2$	
CV	3	$2 = 3^3 - 6\cdot 3^2 + 9\cdot 3 + 2$	$\leftarrow m = 2$
EP	5	$22 = 5^3 - 6\cdot 5^2 + 9\cdot 5 + 2$	$\leftarrow M = 22$

We say

Abs Max is $y = 22$ and occurs @ $x = 5$

Abs min is $y = 2$ and occurs @ $x = 3$

Now we'll do some analysis to help us graph fcns to find this information

① relative maxs & mins

② intervals where curve is rising & falling

③ intervals where curve is smiling & frowning

After looking @ p6 graph we find

① Rel Max of 6 @ $x = 1$

Rel Min of 2 @ $x = 3$

② Curve rising on $(-\infty, 1]$ and $[3, \infty)$

Curve falling on $[1, 3]$.

③ Curve is smiling on $[2, \infty)$

Curve is frowning on $(-\infty, 2]$

Ex Graph $y = f(x) = x^3 - 6x^2 + 9x + 2$
and find all important points & behaviors

$\boxed{1}$ $f'(x) = 3x^2 - 12x + 9 \overset{set}{=} 0$

$3(x^2 - 4x + 3) = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$ are CV

$\boxed{2}$ $f''(x) = 6x - 12 \overset{set}{=} 0$

$6x = 12$

$x = 2$ is a HCV
hypercritical value

p6

	x	y (ht)	y′ (slope)	y″ (mood)
	0			
CV	1	6 dot	0 hor tan	−6 neg ⇒ frown
HCV	2	4 dot	−3	0 inflection pt
CV	3	2 dot	0 hor tan	6 pos ⇒ smile
	4