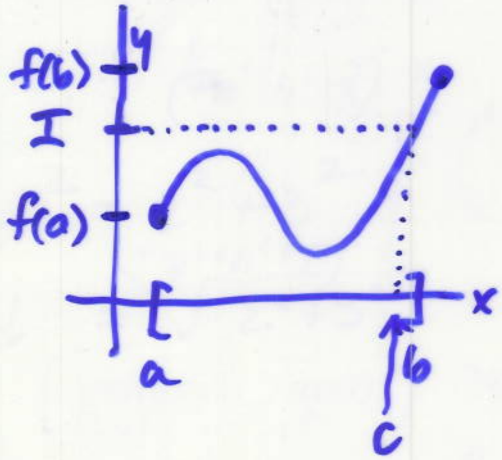
M 191 Lect #13 3-7-11

Recall IVT (Intermediate Value Th)

IVT - Let f be a cont fcn on a closed int [a,b]

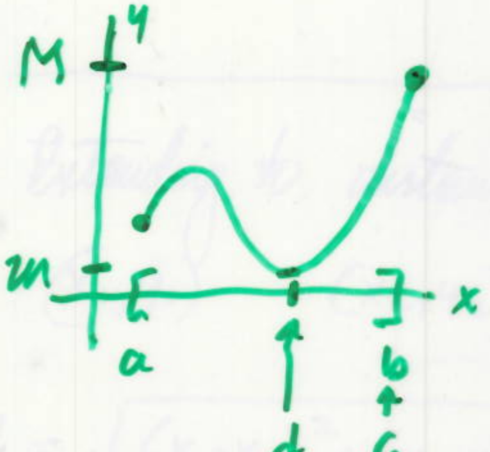


For any y value between f(a) + f(b) there exist $c \in [a,b]$ such that

$f(c) = I$

Now EVT (Extreme Value Theorem)

EVT - Let f be a cont fcn on a closed int [a,b]



There exists a $c \in [a,b]$ such that

$f(c) \geq f(x)$ for all $x \in [a,b]$

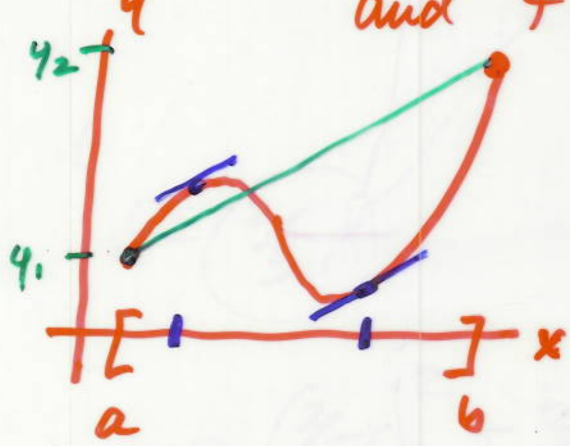
and

There exists a $d \in [a,b]$ such that

$f(d) \leq f(x)$ for all $x \in [a,b]$

MVT – (Mean Value Theorem)

MVT – Let f be cont fcn on a closed int $[a,b]$ and f be differentiable on (c,d).



There exists (at least) a value $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{y_2-y_1}{x_2-x_1}$$

Ex Apply the MVT to the fcn $f(x) = x^3-6x^2+9x+2$ on the interval $[0,5]$. (Find the c)

$f'(x) = 3x^2-12x+9$

$f(5) = 125-150+45+2 = 22$

$f(0) = 2$

$$\frac{f(b)-f(a)}{b-a} = \frac{22-2}{5-0} = 4$$

$f'(c) = 3c^2-12c+9 \overset{\text{set}}{=} 4$

$3c^2-12c+5=0$

$$c = \frac{+12 \pm \sqrt{144-4\cdot3\cdot5}}{2\cdot3}$$

$$= \frac{12\pm\sqrt{84}}{6} = \frac{12\pm2\sqrt{21}}{6}$$

$$= \frac{6\pm\sqrt{21}}{3} \doteq 3.52, .47$$

Both 3.52 & .47 are in the interval. So $c \doteq 3.52$ & $c \doteq .47$ are the two x values guaranteed by the MVT.

<u>Example</u> that has a type II CV and HCV

Find all important characteristics of this fcn and draw its graph

$$y = f(x) = x^{2/3}(x-5) + 1$$

① $f'(x) = x^{2/3} \cdot 1 + (x-5) \cdot \frac{2}{3} x^{-1/3} \cdot 1 \overset{\text{set}}{=} 0$

$$= x^{2/3 - 1 + 1} + x^{-1/3} \cdot 2/3\,(x-5) = 0$$

$$= x^{-1/3 + 1} + x^{-1/3} \cdot 2/3\,(x-5) = 0$$

$$= x^{-1/3} \cdot x^{1} + x^{-1/3} \cdot 2/3\,(x-5) = 0$$

$$= x^{-1/3}\left(\frac{3}{3}x + 2/3\,(x-5)\right) = 0$$

$$= \frac{1}{3} x^{-1/3}\left(3x + 2(x-5)\right) = 0$$

$$= 1/3\, x^{-1/3}\,(5x - 10) = 0$$

$$= \frac{5}{3}\,\frac{x-2}{x^{1/3}} = 0$$

① $f'(x) = \frac{5}{3} \frac{x-2}{x^{1/3}} \overset{set}{=} 0 \Rightarrow x-2=0$

$x=2$ Type I CV.

② $f'(x) = \frac{5}{3} \frac{x-2}{x^{1/3}} \overset{set}{=} DNE \Rightarrow x=0$ Type II CV

$$f'(x) = \frac{5}{3}\,\frac{x-2}{x^{1/3}}$$

$$\tfrac{1}{3} - 1 = -\tfrac{2}{3}$$

$$f''(x) = \frac{5}{3}\,\frac{x^{1/3}\cdot 1 - (x-2)\cdot \frac{1}{3}x^{-2/3}}{x^{2/3}}$$

$$= \frac{5}{3}\,\frac{x^{\frac{1}{3}-1+1} - (x-2)\,\frac{1}{3}\,x^{-2/3}}{x^{2/3}}$$

$$= \frac{5}{3}\,\frac{x^{-2/3}\cdot x^{1} - \frac{1}{3}(x-2)\,x^{-2/3}}{x^{2/3}}$$

$$= \frac{5}{3}\,\frac{x^{-2/3}\left(\frac{3}{3}x - \frac{1}{3}(x-2)\right)}{x^{2/3}}$$

$$= \frac{5}{9}\,\frac{x^{-2/3}\left(3x - x + 2\right)}{x^{2/3}}$$

$$= \frac{5}{9}\,\frac{x^{-2/3}\left(2x+2\right)}{x^{2/3}}$$

$$= \frac{10}{9}\,\frac{x+1}{x^{4/3}}$$

$f''(x) = \frac{10}{9} \cdot \frac{x+1}{x^{4/3}} \begin{cases} \overset{set}{=} 0 \\ \overset{set}{=} DNE \end{cases}$ P 6

$x = -1$ HCV Type I

$x = 0$ HCV Type II

	x	y	y'	y''
	-2			neg neg/pos
HCV	-1	-5	5	0
CV HCV	0	1	undef	undef
CV	2	-3	0 hor. tan	1.3