M 191 Lect #14 3-8-11

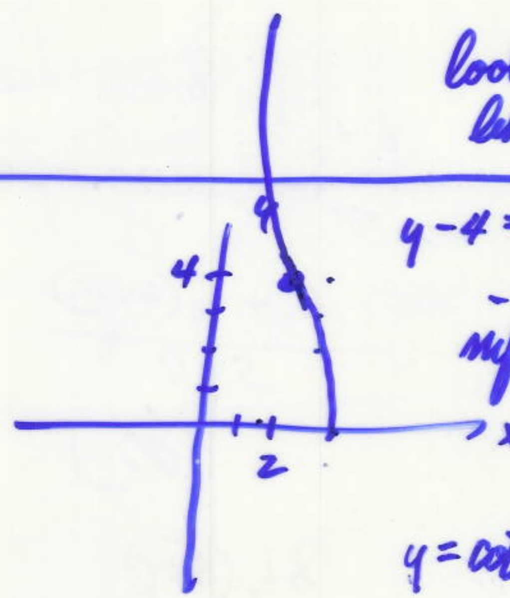
We want to look at some situations which resemble the 7 major behaviors of fcns.

1. smooth
2. corner
3. cusp
4. vert tan
5. jump
6. empty hole
7. vert asy

$y = x^3$

$y = -x^3$

x	y	y'	y''	
HCV 2	4	-2	pos. 0 neg	infl pt



looks a little like

$y - 4 = -(x-2)^3$

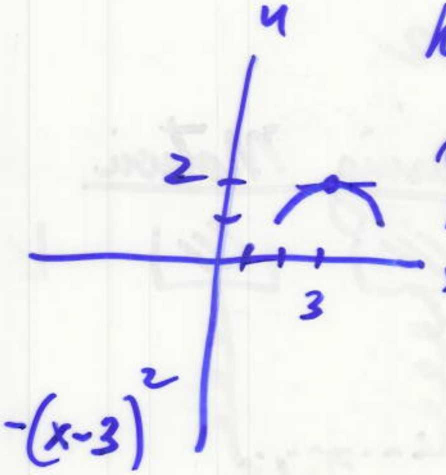
infl pt

$y = \cot x$

or

$y = -x^3$

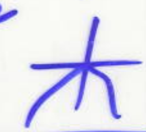
x	y	y'	y''
3	2	0	-1 frown



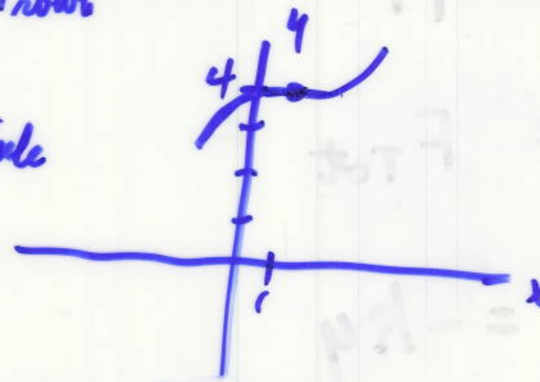
hump
rel
max

looks like $y-2=-(x-3)^2$

$y=x^2$ 

$y=-x^2$ 

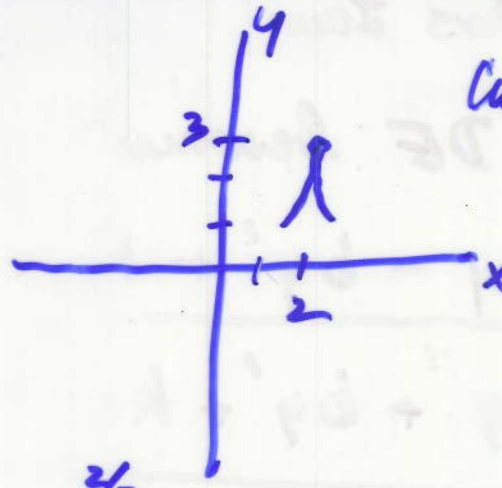
	x	y	y'	y''
CU HCV	1	4	0	neg frown 0 pos smile



shelf

looks like $y-4=(x-1)^3$

x	y	y'	y''
2	3	undef **	pos undef ** pos



Cusp

** one sided limits are infinite

$y=$

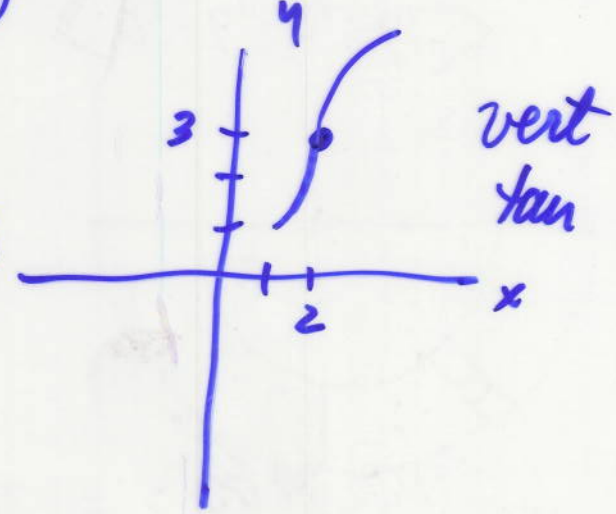
$y-3=-(x-2)^{2/3}$

x	y	y′	y″
2	3	undef **	pos undef ** neg

cont @ 2

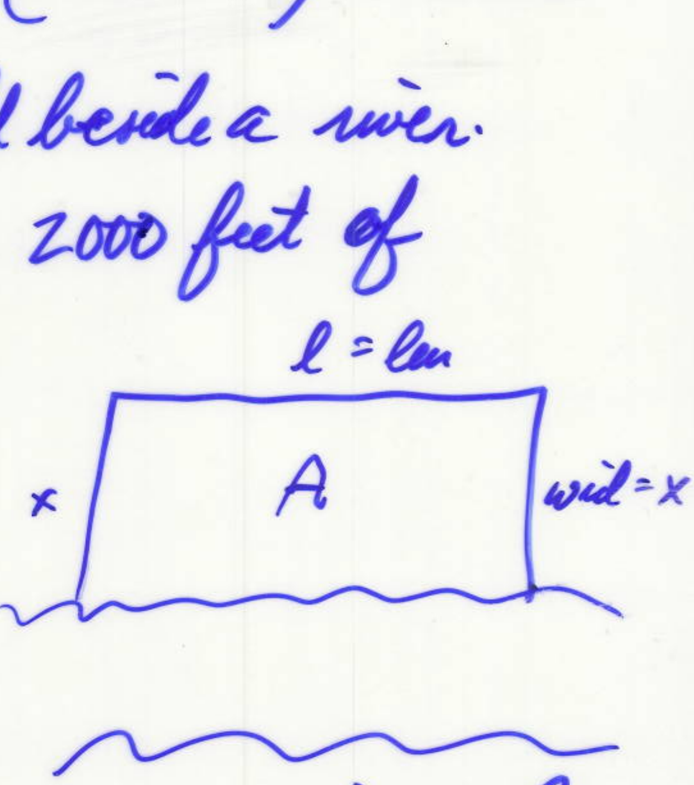
looks like

$$y-3 = (x-2)^{1/3}$$



Optimization (Max/Min) (extrema) Problems

Ex A rich lady has a field beside a river. A richer friend gives her 2000 feet of left over fence. Find the dimensions of the field of greatest contained area.



Let A = area enclosed — Maximize A

A = len × wid

$A = lx$

objective function

Constraint is
$2x + l = 2000$

$l = 2000 - 2x$

$A = (2000 - 2x) \cdot x = 2000x - 2x^2$

$\frac{dA}{dx} = 2000 - 4x \overset{\text{set}}{=} 0$

$2000 = 4x$

$x = 500$ TI CV

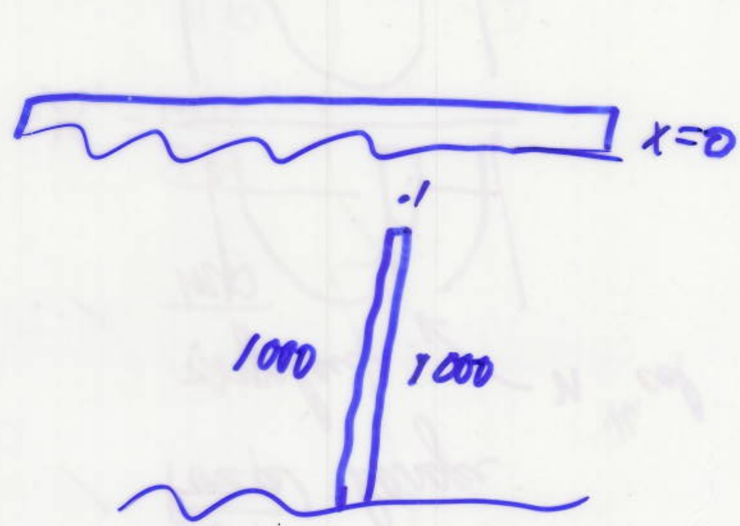
no TII CV

dom for x var

is $(0, 1000)$

fcn works on a closed

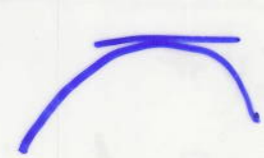
$[0, 1000]$



$0 < x < 1000$

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\frac{dA}{dx} = \frac{d}{dx}(2000-4x) = -4$$

	x	A	A'	A''
	0	0		
CV	500	500,000	0 hor tan	-4 frown
	1000	0		



So
Rel. Max A when $x = 500$ ft

$$\begin{aligned} l &= 2000 - 2x \\ &= 2000 - 2 \cdot 500 \\ l &= 1000 \end{aligned}$$

$$\begin{aligned} A &= 2000x - 2x^2 \\ &= 2000 \cdot 500 - 2(500)^2 \\ &= 500{,}000 \end{aligned}$$

The dim of the largest field are 1000 long by 500 ft ($\perp$ to river).

Infinite limits and limits at infinity

$$\lim_{x \to 2} \frac{1}{(x-2)^3}$$

$$\lim_{x \to \infty} \frac{x^2-1}{x^2+1}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = 1$$

$$\lim_{x \to \infty} \left(\sqrt{x+1} - \sqrt{x}\right) \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

used conjugate