M192 Lect #15 3-14-11

Position, Velocity, acceleration problem

$s = t^3 - 6t^2 - 15t + 10$

$s' = v = 3t^2 - 12t - 15$

$s'' = a = 6t - 12$

Question	t	pos s	vel $s' = v$	accel $s'' = a$
1. How fast is obj moving initially	0		—	
2. When will vel be zero?	—		0	
3. Where will vel be zero		—	0	
4. What is accel at $t=2$?	2			—
5. When is accel zero?	—			0
6. When is vel max or min	2	--		0
7. How high is vel max ht		—	0	neg

$a = 6t - 12 \stackrel{set}{=} 0$

$t = 2$

Last time we did the Largest Field Problem

Now we do the <u>Largest open box Problem</u>

i.e., another Optimization Problem

Suppose we are to build an open rect box (no top) using 900 ft^2 of material and we want the largest box possible. The length is 3 times the width.

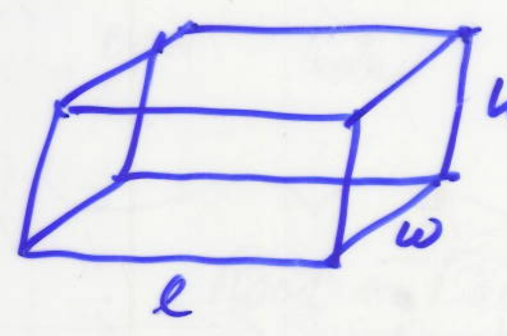
Obj. fcn.: Maximize $V = lwh$

Constraints: $l = 3w$

$$900 = lh + lw + wh + lh + wh$$

$$900 = 2lh + 2wh + lw$$

l = length
w = width
h = height



Plug $l = 3w$ in everywhere

$V = 3wwh$ and $900 = 2 \cdot 3wh + 2wh + 3ww$

$$900 = 8wh + 3w^2$$

Solve for h

$$\frac{900-3w^2}{8w} = \frac{\cancel{8w}h}{\cancel{8w}}$$

$$h = \frac{900-3w^2}{8w}$$ Plug into V's formula

$$V = 3w^{\cancel{2}} \, \frac{900-3w^2}{8\cancel{w}} = \frac{3}{8}\left(900w - 3w^3\right)$$

Looks like $y = f(x)$ $\frac{dy}{dx} \overset{set}{=} 0$

$$\frac{dV}{dw} \overset{set}{=} 0$$

$$\frac{dV}{dw} = \frac{3}{8}\left(900 - 9w^2\right) \overset{set}{=} 0$$

$$100 - w^2 = 0$$

$$w^2 = 100$$

$w = \pm 10$ TI CV

$w = 0$ TII CV

w needs to be pos so we abandon the −10

Is the CV $w = 10$ a rel max
rel min or shelf

$$\frac{dV}{dw} = \frac{3}{8}\left(900 - 9w^2\right)$$

$$= \frac{27}{8}\left(100 - w^2\right)$$

$$\frac{d^2V}{dw^2} = \frac{27}{8}(-2w) = -\frac{27}{4}w$$

Plug $w = 10$ into $\frac{d^2V}{dw^2}$

CV

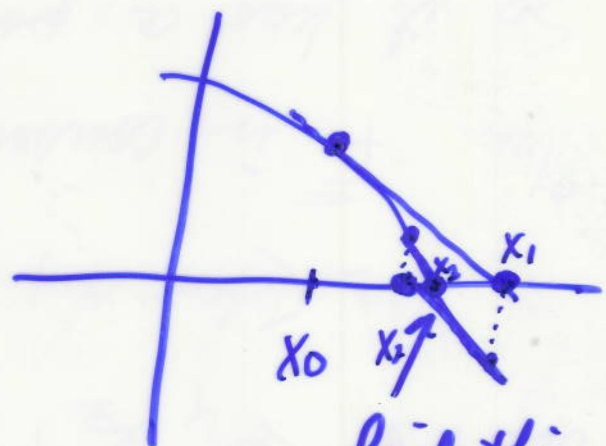
$$\frac{d^2V}{dw^2} = -\frac{27}{4}(10) = -\frac{135}{2}$$ is neg $\Rightarrow$ frown
$\Rightarrow$ rel max

	w	V	V'	V''
CP	10	......	0	$-\frac{135}{2}$ neg
	0	0		

The dims of the largest open box is
$l = 30$ ft $w = 10$ ft $h = \frac{900 - 3(10)^2}{8 \cdot 10} = \frac{600}{80} = \frac{15}{2} = 7.5$ ft
Vol $= 30 \times 10 \times 7.5 = 2250$ ft^3

p5

Newton's Method for find roots (or zeros) of a function.

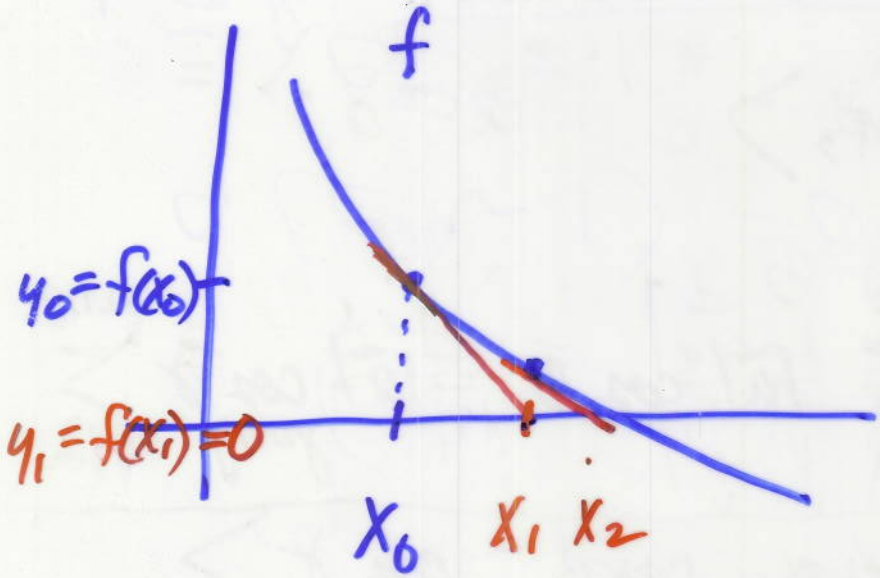


find this crossing x-value to 10 decimal places

Start as some x_0 value

We will slide down the tangent line to get to x_1

Repeat until all the rest of the x's are the same



$$m_{tan} = f'(x_0) = \frac{rise}{run} = \frac{\overset{0}{y_1} - y_0}{x_1 - x_0} = \frac{-y_0}{x_1 - x_0}$$

$$(x_1 - x_0)\frac{\cancel{f'(x_0)}}{\cancel{f'(x_0)}} = \frac{-y_0}{\cancel{x_1 - x_0}} \frac{\cancel{(x_1 - x_0)}}{f'(x_0)}$$

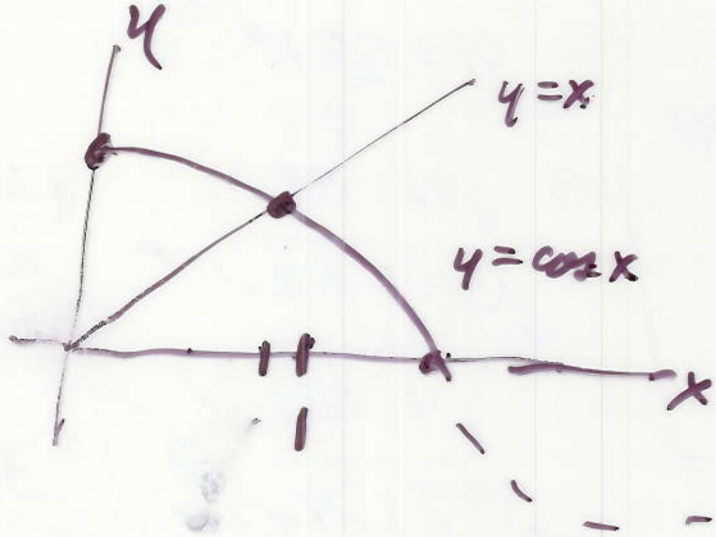
$$x_1 - x_0 = -\frac{y_0}{f'(x_0)} = -\frac{f(x_0)}{f'(x_0)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

Ex. Find where $y = x$ crosses $y = \cos x$

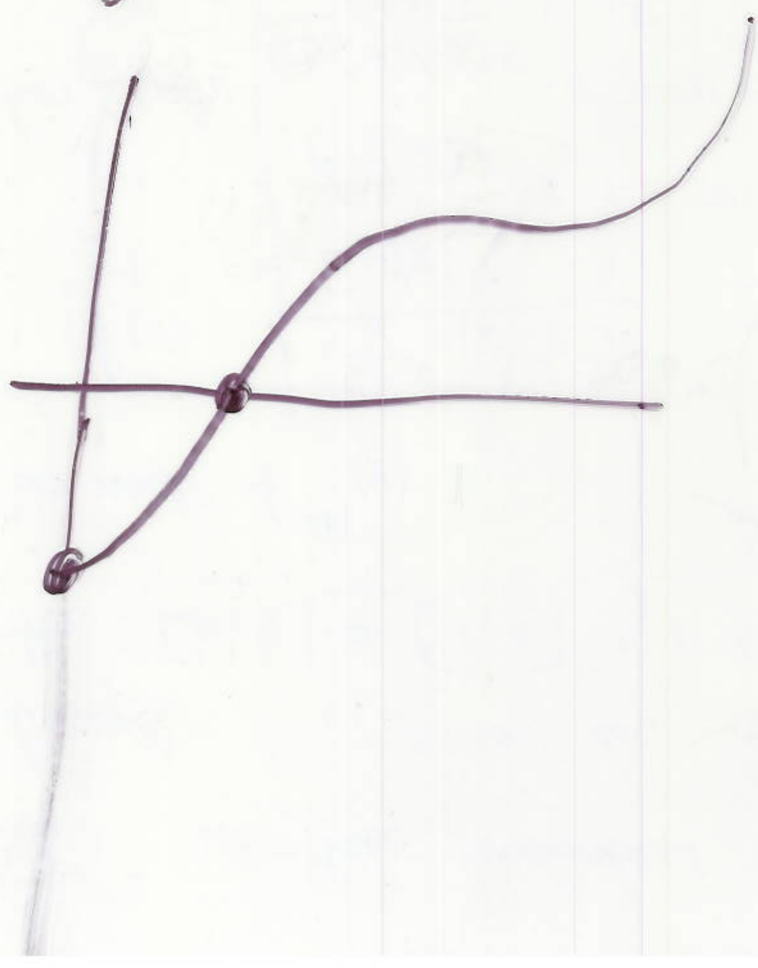


I want $x = \cos x$

I want $x - \cos x$ to $= 0$

So we let $f(x) = x - \cos x$

and find where $f(x) = 0$



$$f(x) = x - \cos x$$

$$f'(x) = 1 + \sin x$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x)}$$

$$x_1 = x_0 - \frac{(x_0 - \cos x_0)}{(1 + \sin x_0)}$$

Let's choose $x_0 = 1$

$$x_1 = 1 - \frac{(1 - \cos 1)}{(1 + \sin 1)} = .70155$$

real 1