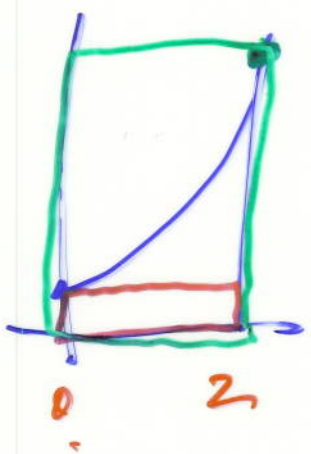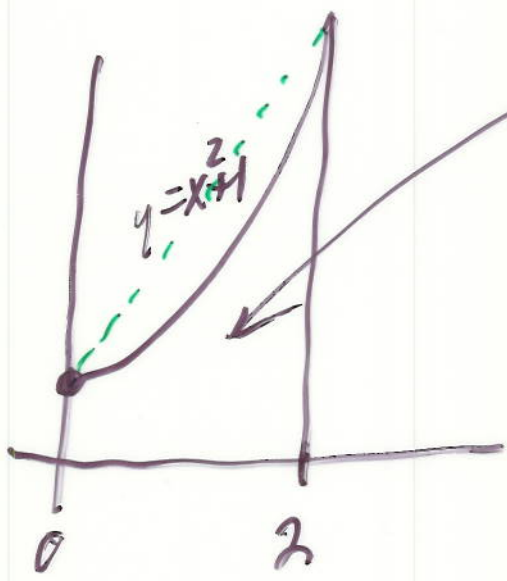We now attempt the second major problem
in Calculus : Find the area under the curve
Given $y = f(x) = x^2 + 1$. Find area A here

$y = x^2 + 1$

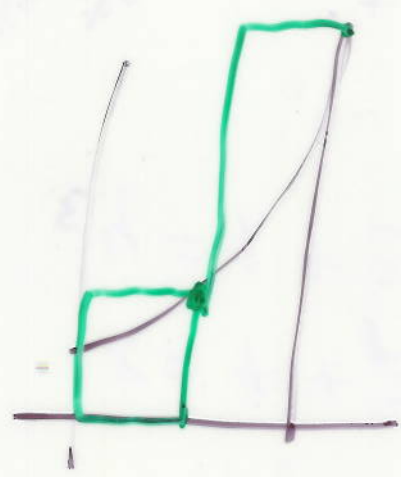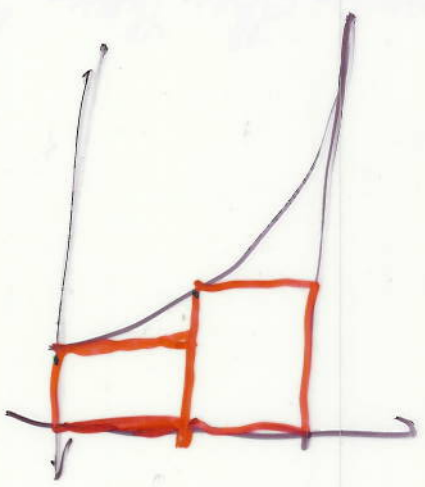0   2

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0   2

$$L_1 \leq A \leq U_1$$

$$\text{base} \cdot \text{ht} \leq A \leq \text{base} \cdot \text{ht}$$

$$2 \cdot 1 \leq A \leq 2 \cdot 5$$

$$2 \leq A \leq 10$$

Guess $A \approx 6$

$$y = x^2 + 1$$

$$\uparrow$$
$$1$$
$$\uparrow$$
$$x^2 + 1$$

$$5$$

$$L_2 \leq A \leq U_2$$

$$1 \cdot 1 + 1 \cdot 2 \leq A \leq 1 \cdot 2 + 1 \cdot 5$$

$$3 \leq A \leq 7$$

$$\text{Guess } A \doteq \frac{7+3}{2}$$

$$A \doteq 5$$