M 191 Lect #20 4-6-11

Fundamental Theorem of "All" Calculus, Part 2.

(FTC2)

Let f be a cont. fcn on [a,b]

Let F be any antiderivative of f

Then

$$\int_a^b f(x)\,dx \overset{\text{FTC2}}{=} F(x)\Big]_{x=a}^{b} = F(b) - F(a)$$

$$\int_a^b f(x)\,dx = \int f(x)\,dx\,\Big]_a^b$$

Evaluate $\int_{-2}^{3}(8x^3+12x^2-6x+7)\,dx$ p2

FTC2

$= 8\cdot\frac{x^4}{4} + 12\frac{x^3}{3} - 6\frac{x^2}{2} + 7x + C\Big]_{x=-2}^{3}$

$= 2x^4 + 4x^3 - 3x^2 + 7x + C\Big]_{-2}^{3}$

$= (2\cdot 81 + 4\cdot 27 - 3\cdot 9 + 7\cdot 3 + C) - (\quad)$

$= (162 + 108 - 27 + 21 + \cancel{C}) - (+32 - 32 - 12 - 14 + \cancel{C})$

$= 290$

or

$= 2(81-16) + 4(27-(-8)) - 3(9-4) + 7(3-(-2))$

$= 2(65) + 4(35) - 3(5) + 7(5)$

$= 130 + 140 - 15 + 35 = 290$

Use these rules to evaluate {approximate} an integral when given only numerical data

f

x	2.0	2.3	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.7
y	7	5	4	5	6	8	10	13	12	15

Approximate $\int_{2.0}^{4.4} f(x)\,dx$ using $n = 4$ subdivisions

$$\frac{4.4 - 2.0}{4} = \frac{2.4}{4} = .6$$

$$R_4 = (4 + 6 + 10 + 12)(.6) = 32(.6) = 19.2$$

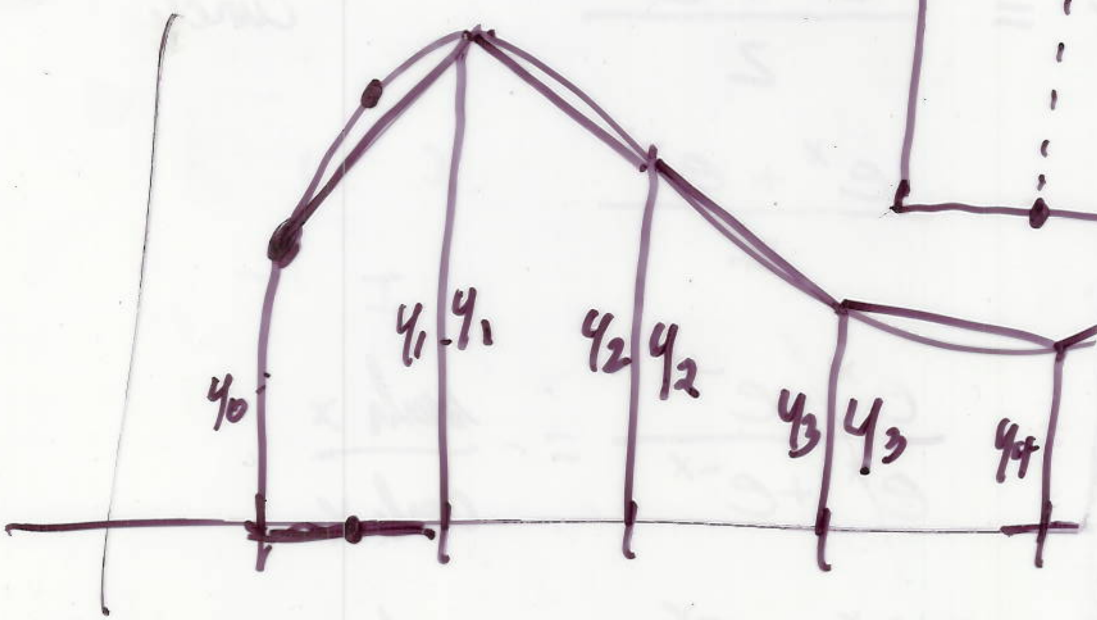
$$L_4 = (7 + 4 + 6 + 10)(.6) = 27(.6) = 16.2$$

$$U_4 = (7 + 6 + 10 + 13)(.6) = 36(.6) = 21.2$$

$$Low_4 = (4 + 4 + 6 + 10)(.6) = 24(.6) = 14.4$$

$$M_4 = (5 + 5 + 8 + 13)(.6) = 31(.6) = 18.6$$

P4

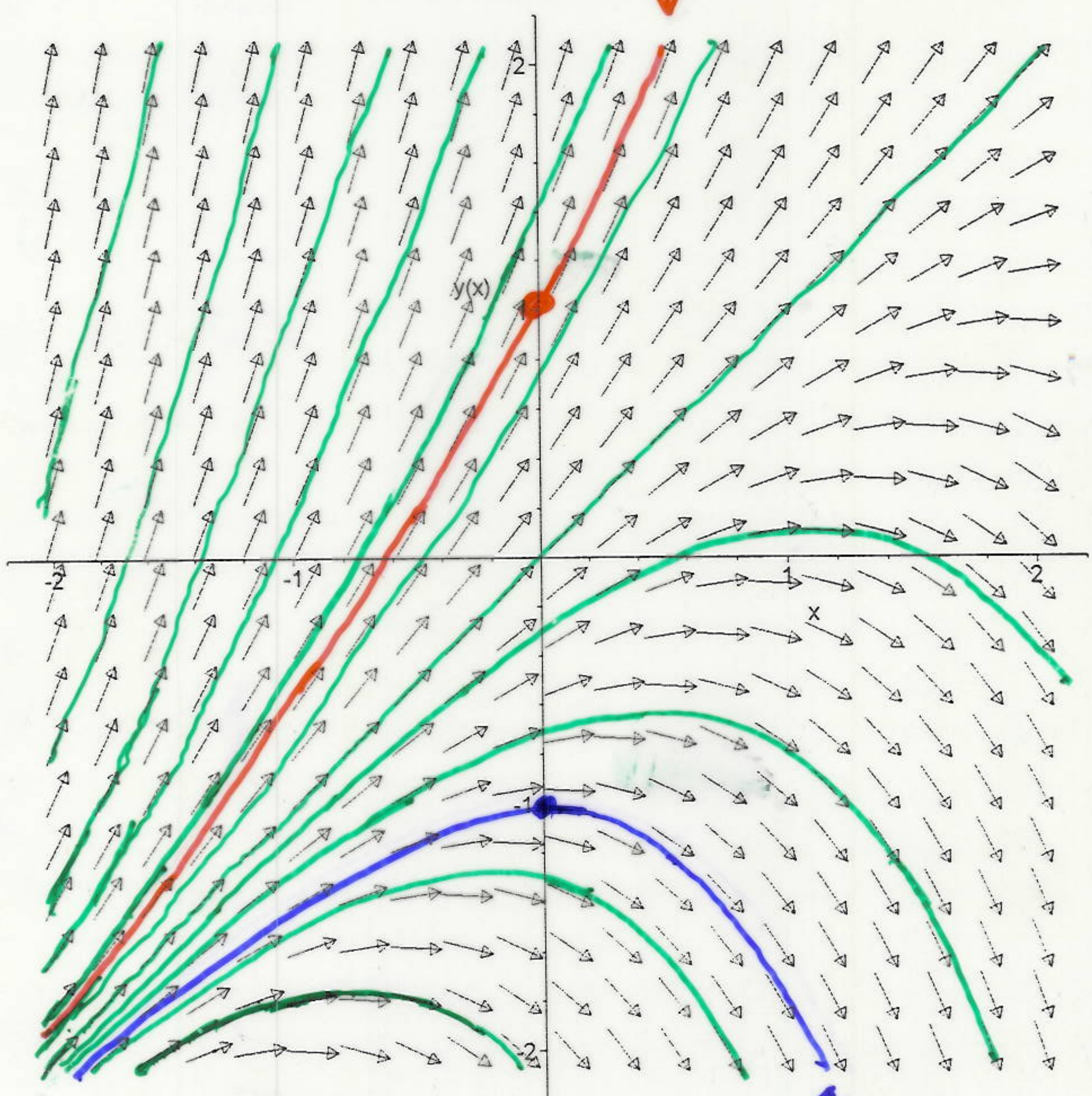


$$\frac{y_0 + y_1 + y_1 + y_2 + y_2 + y_3 + y_3 + y_4}{2}$$

$$T_4 = \left(\frac{y_0 + 2y_1 + 2y_2 + 2y_3 + y_4}{2}\right)\Delta x$$

Soln that satisfies $y(0) = 1$

Slope Field for y'=1-x+y



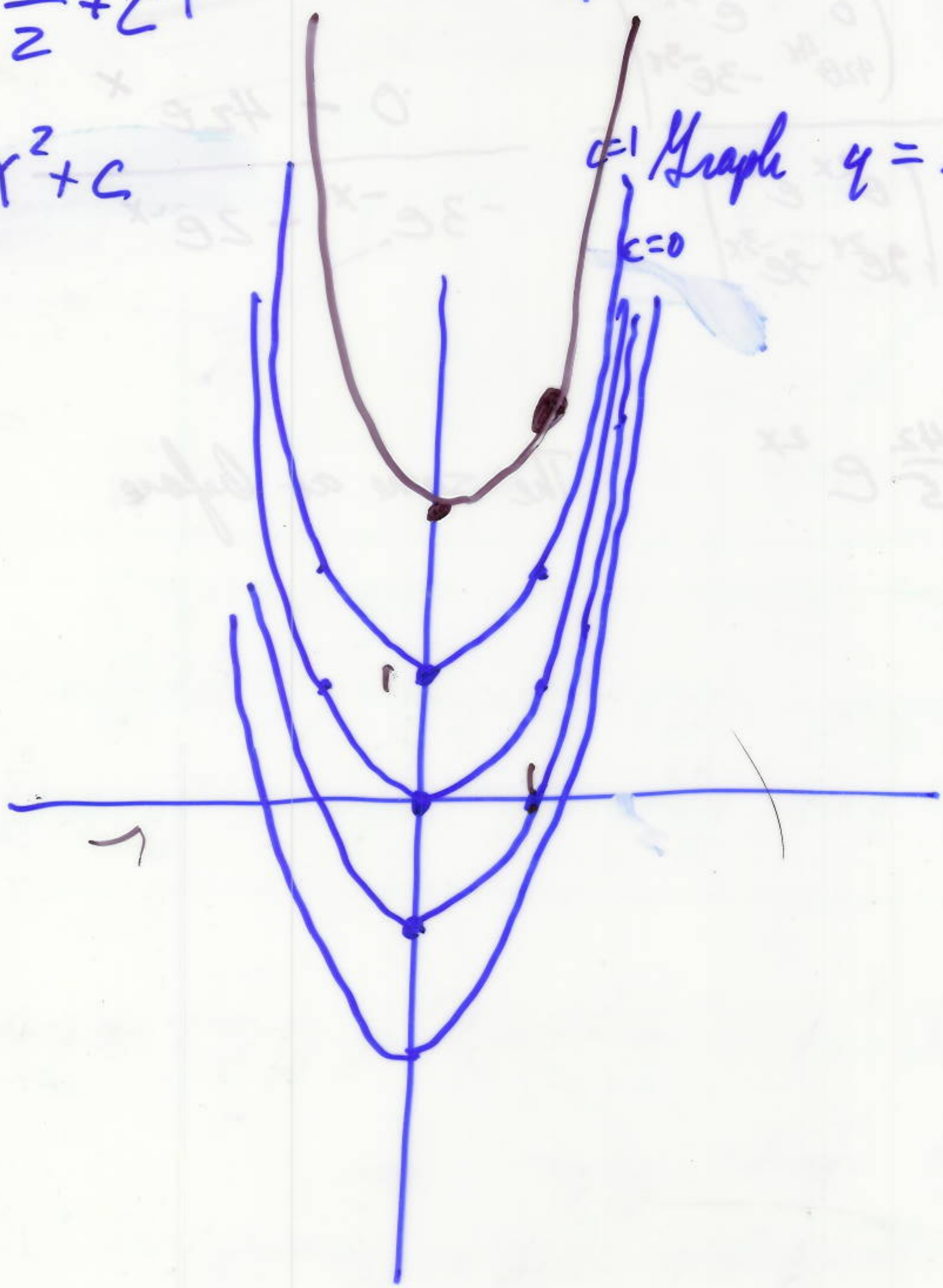
Solution that satisfies $y(0) = -1$

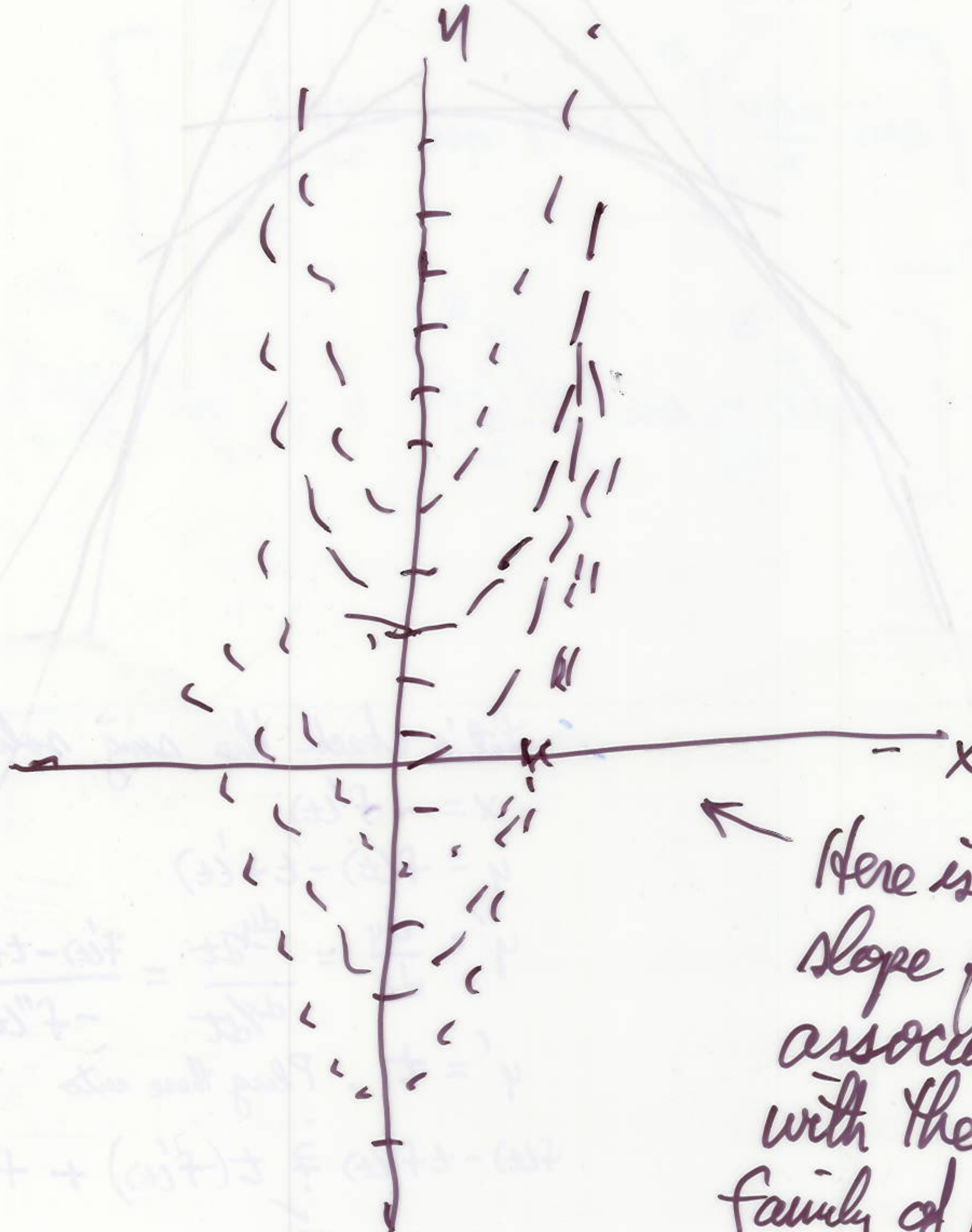
Why do we do slope fields?

F	f
$2 \cdot \frac{x^2}{2} + C$	$2x + 0$

$x^2 + C$

$c=1$ Graph $y = x^2 + c$





Here is the slope field associated with the previous family of parabolas