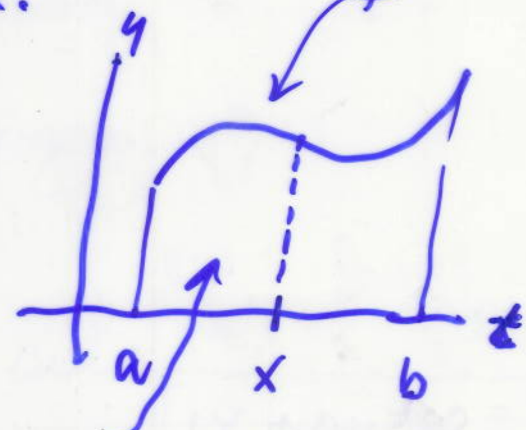
M 191 Lect #21 4-11-11

Here is how the FTC1 works.

$$A = \int_a^x f(t)\,dt$$

this area



$$\frac{dA}{dx} = \frac{d}{dx}\int_{t=a}^{t=x} f(t)\,dt = f(x)$$

Ex $\frac{d}{dx}\int_2^x \cos^3(t)\,dt = \cos^3(x)$

$$\frac{d}{dx}\int_{-5}^x \sqrt{t^3+1}\,dt = \sqrt{x^3+1}$$

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let $g(x) = \int_0^x 4^{t+5}\,dt$

$$g'(x) = 4^{x+5}$$

F	F'
F	f

$$\frac{d}{dx}\int_{x^2}^{\sin x} f(t)\,dt \overset{\text{FTC2}}{=} \frac{d}{dx}\; F(t)\Big]_{t=x^2}^{t=\sin x}$$

$$= \frac{d}{dx}\Big(F(\sin x) - F(x^2)\Big)$$

$$= F'(\sin x)\cdot\cos x - F'(x^2)\cdot 2x$$

$$= f(\sin x)\cdot\cos x - f(x^2)\cdot 2x$$

$$\frac{d}{dx}\int_{3x}^{x^4} \tan(2t)\,dt$$

$$= \tan\big(2(x^4)\big)\cdot 4x^3 - \tan\big(2(3x)\big)\cdot 3$$

$$\frac{d}{dx}\int_a^x t^5\,dt$$

$$= x^5\cdot 1 - a^5\cdot 0$$

$$= x^5$$

for phun.

Fundament Theorem of (all) Calculus

Let f be a cont fcn on $[a,b]$

Part I: If $g(x) = \int_a^x f(t)\,dt$

Then g is cont on $[a,b]$

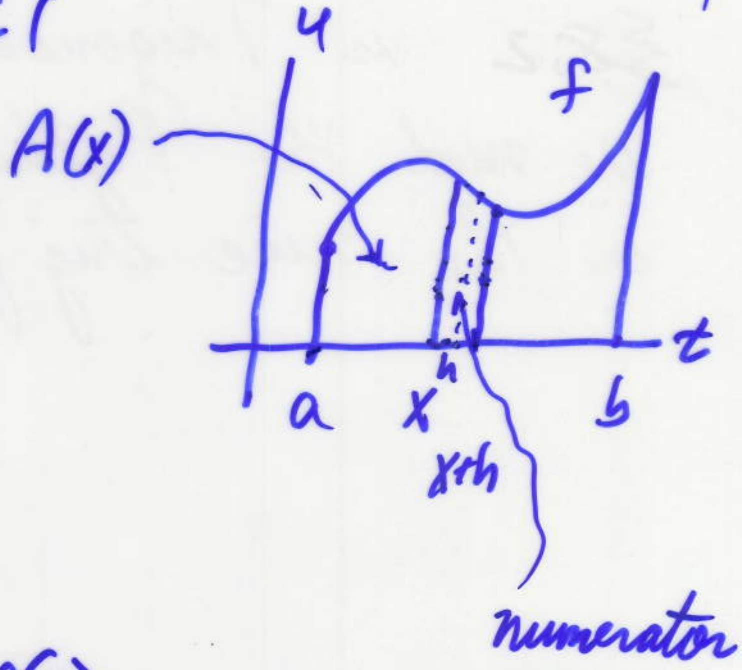
g' is cont on (a,b)

$g'(x) = f(x)$

Part II If F is any antiderivative of f

Then $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

A loose proof of FTC I



$$A(x) = g(x) = \int_a^x f(t)\,dt$$

Find $g'(x)$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\text{area under curve between } x \text{ \& } x+h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h} \cdot f(x)}{\cancel{h}} = f(x)$$

Substitution (For anti-diff. of harder fcns

Let u = inside of most complicated

$$\int (5x+3)^2\,dx$$

let $u = 5x+3$

$du = 5\,dx$

$dx = \frac{du}{5}$

sort of looks like

$$= \int u^2\,dx$$

$$= \int u^2\,\frac{du}{5}$$

$$= \frac{1}{5}\int u^2\,du$$

$$= \frac{1}{5}\,\frac{u^3}{3} + C$$

$$= \frac{1}{15}(5x+3)^3 + C,$$

Check

$$\frac{d}{dx}\left(\frac{1}{15}(5x+3)^3 + C\right)$$

$$= \frac{1}{15}\cdot 3(5x+3)^2 \cdot 5 + 0$$

$$= (5x+3)^2$$

2 piece subst

let $u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}\int \sqrt{x^2+9}\,2x\,dx$

$= \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C_1$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} (x^2+9)^{3/2} + C$

$= \frac{1}{3} \sqrt{x^2+9}^{\,3} + C$

4 piece subst

p8

$$\frac{1}{2}\int_0^2 (x^2+1)^4 \, 2x \, dx$$

$$\left[\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \\ x=0 \Rightarrow u=1 \\ x=2 \Rightarrow u=5 \end{array}\right.$$

$$= \frac{1}{2}\int_{u=1}^{5} u^4 \, du = \frac{1}{2}\int_{x=0}^{2} u^4 du$$

$$= \frac{1}{2} \left. \frac{u^5}{5} \right]_{u=1}^{5} = \frac{1}{2} \left. \frac{(x^2+1)^5}{5} \right]_0^2$$

$$= \frac{1}{10}\left[5^5 - 1^5\right]$$

$$= \frac{1}{10}\left[3124\right]$$

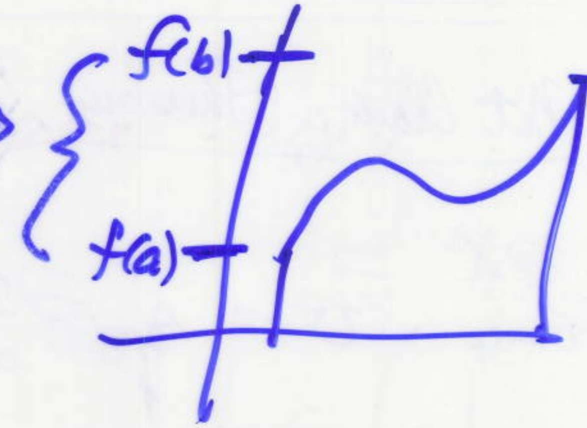
$$= 312.4$$

s	s'
s	v
F	f
f	f'
F	f

$$\int_a^b f'(t)\,dt$$

$$= f(t)\Big]_a^b = f(b) - f(a)$$

= net change in the fcn.



$\int$ rate d den

$$\frac{ft}{\cancel{sec}} \times \cancel{sec}$$

s	v	$\int$

$$\int v\,dt = s + c$$

$$\int_0^1 (6t+2)\,dt = 3t^2+2t\Big]_0^1 = 3+2-0-0 = 5\text{ ft}$$

= net change in position

$y' = x^2 - 2x \quad , \quad y(0) = 1$