M191 Lect #23 4-18-11

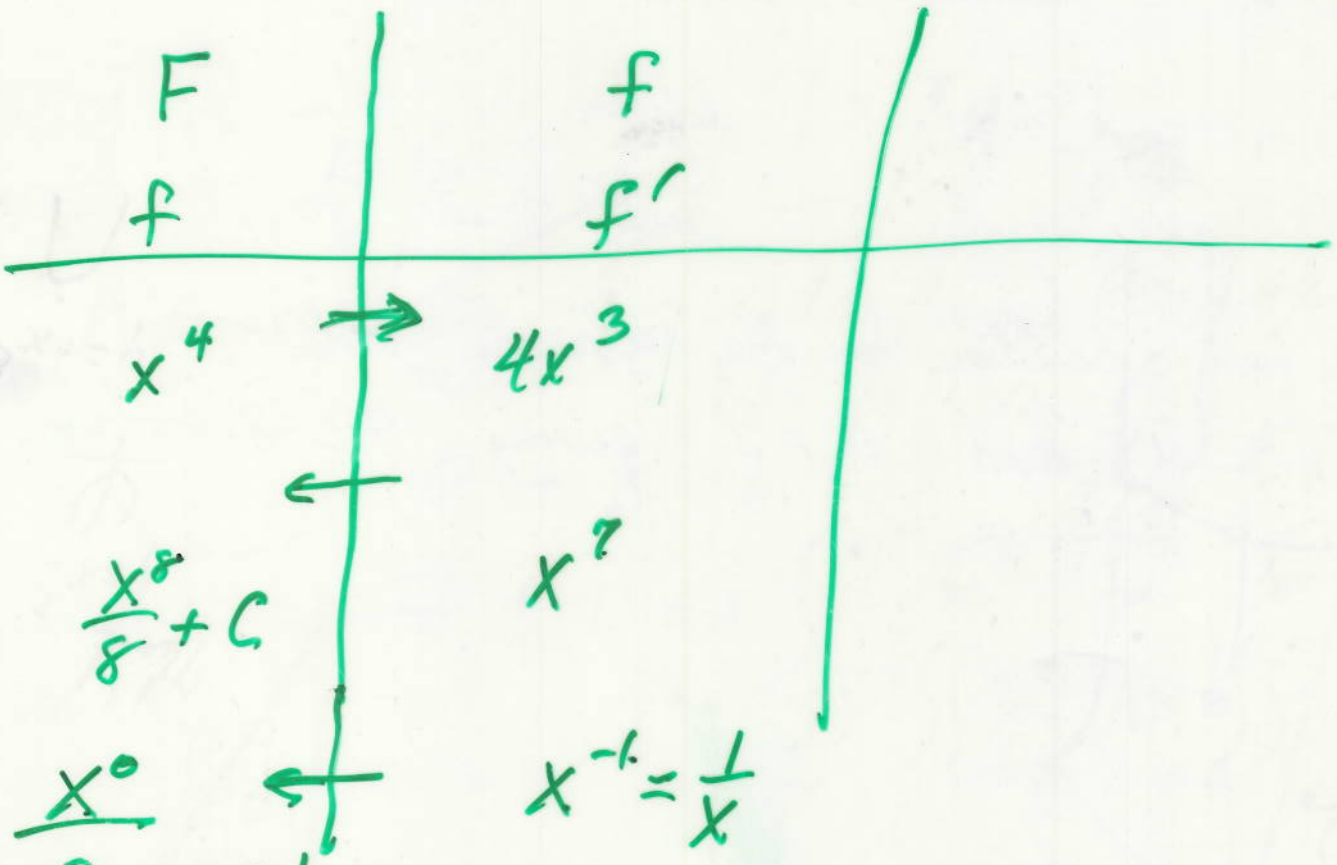
FTC2 If f is cont on $[a,b]$ and if $F' = f$

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big]_a^b$$

$$= \int_a^b f(t)\,dt$$

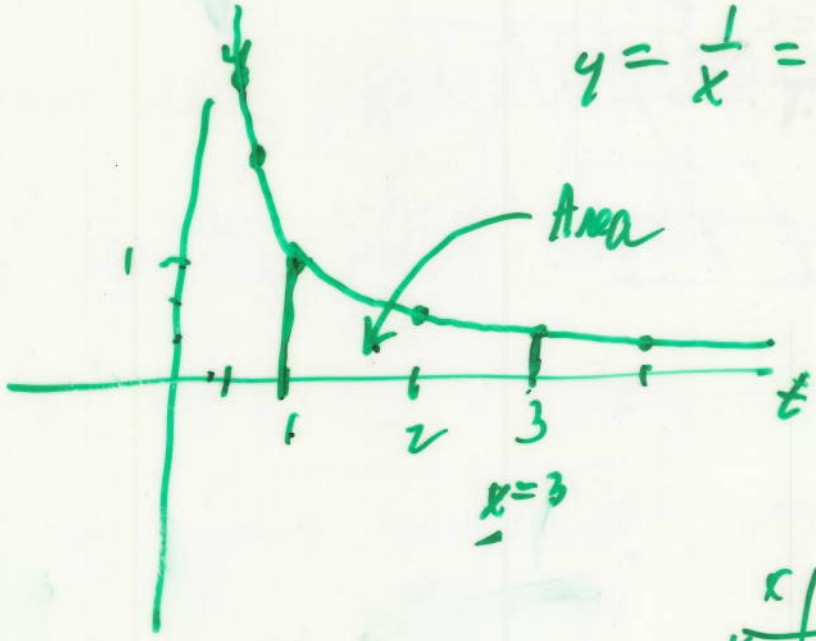
FTC1 $$g(x) = \int_a^x f(t)\,dt$$

$$g'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



Undefined

But $x^{-1}=\frac{1}{x}$ does have a graph



$$\text{Area} = \int_1^x \frac{1}{t}\,dt = \ln(x)$$

lon

x	y
1/3	3
1/2	2
1	1
2	1/2
3	1/3

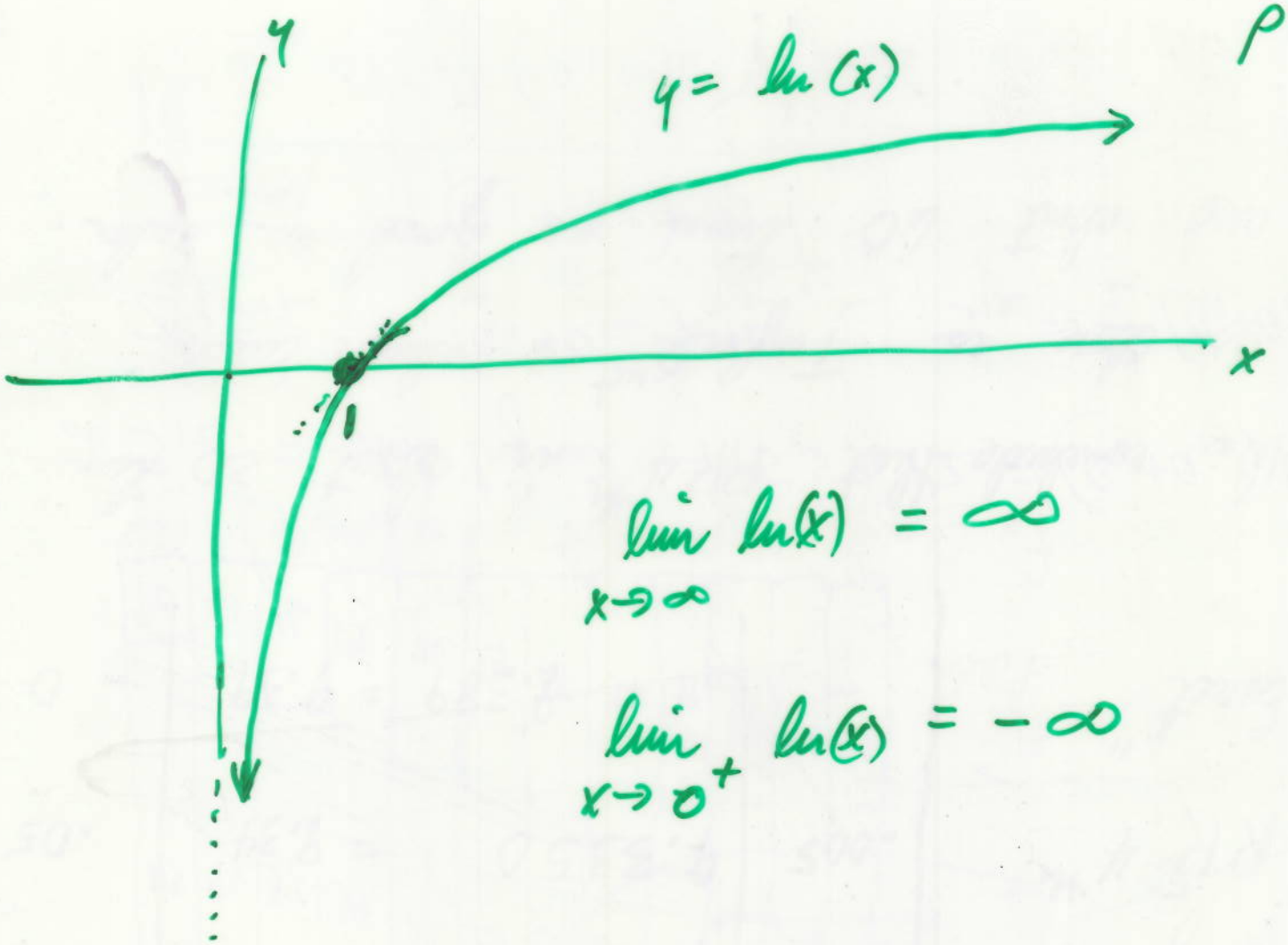
Find $\frac{d}{dx} \ln(x) = g(x)$

$$\frac{d}{dx} \ln(x) = \frac{d}{dx} \int_1^x \frac{1}{t}\, dt = \frac{1}{x}$$

$$\frac{d^2}{dx^2} \ln x = \frac{d}{dx} \frac{1}{x} = \frac{d}{dx} x^{-1} = -1 x^{-2}$$

$$= -\frac{1}{x^2}$$

x	$\ln(x)$ y	$\frac{1}{x} = y'$	$-\frac{1}{x^2} = y''$
1	$\ln(1) = 0$ on x axis	$\frac{1}{1} = +1$ rising	$-\frac{1}{1^2} = -1$ frown
x on $(0, \infty)$	?	pos rises	neg frowns



$$\lim_{x \to \infty} \ln(x) = \infty$$

$$\lim_{x \to 0^+} \ln(x) = -\infty$$

The chain rule for ln(x)

The chain rule for the power fun was

$$y = (3x^2+5x)^8$$

$$\frac{dy}{dx} = 8(3x^2+5x)^7 \cdot (6x+5)$$

$$y = u^8 \qquad u = u(x)$$

$$\frac{dy}{dx} = 8u^7 \cdot \frac{du}{dx}$$

$$y = f(u)$$

$$\frac{dy}{dx} = f'(u) \cdot \frac{du}{dx}$$

$$y = \ln(u)$$

← Chain rule for ln

$$\boxed{\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}}$$

$$\frac{d}{dx} \ln(4x^3 + 5x^2)$$

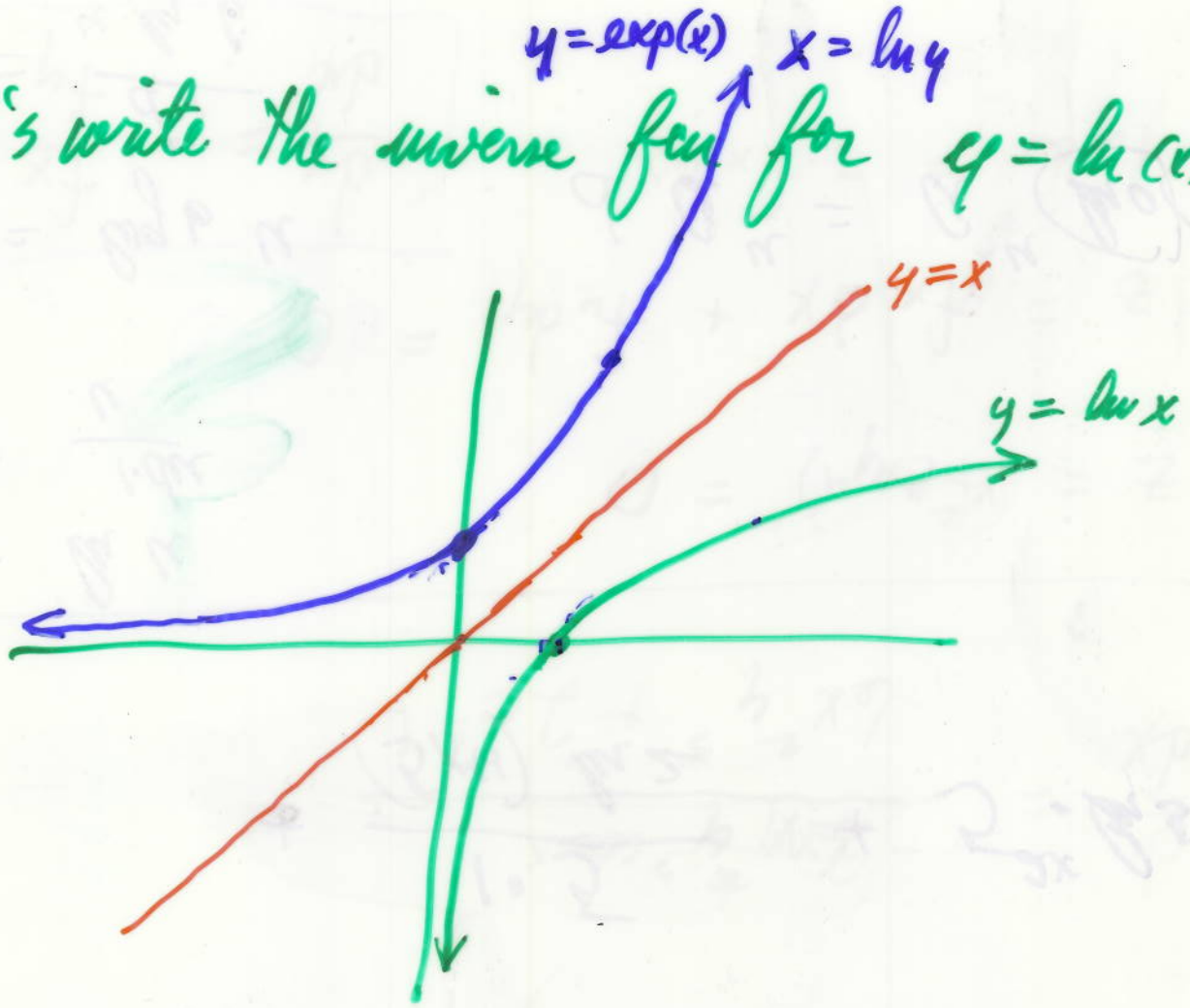
$$= \frac{1}{4x^3 + 5x^2} \cdot (12x^2 + 10x)$$

$$= \frac{2x(6x+5)}{x^2(4x+5)} = \frac{2(6x+5)}{x(4x+5)}$$

$$\frac{d}{dx} \ln(\sin(x)) = \frac{\cos x}{\sin x} = \cot x$$

$$\int \cot x \, dx = \ln(\sin(x)) + C$$

Let's write the inverse for $y = \ln(x)$



$x = \ln y$ means $y = \exp(x)$