M191 Lect #24 4-20-11

We need to see what e is on the graph

We showed this on prev lect p7.

Now we show that $e^x = \exp(x)$

In algebra a^r is defined for all $a>0$ and r rational meaning $r = \frac{m}{n}$ for m & n integers, $n \neq 0$

So e^r = defined for r rat.

$\ln(e^r) = r\ln(e) = r \cdot 1 = r$

$\ln(\exp(r)) = r$. So $\ln(e^r) = \ln(\exp(r))$

Since ln is a 1-1 fcn $e^r = \exp(r)$

But $\exp(x)$ is defined for all real x.

So we define $e^x = \exp(x)$ for the real x's.

Let's do the calculus for this new $\exp(x) = e^x$ fcn.

$y = e^x = \exp(x)$ means $\ln y = x$

Find $\frac{dy}{dx}$

$$\frac{d}{dx} \ln y = \frac{d}{dx} x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1$$

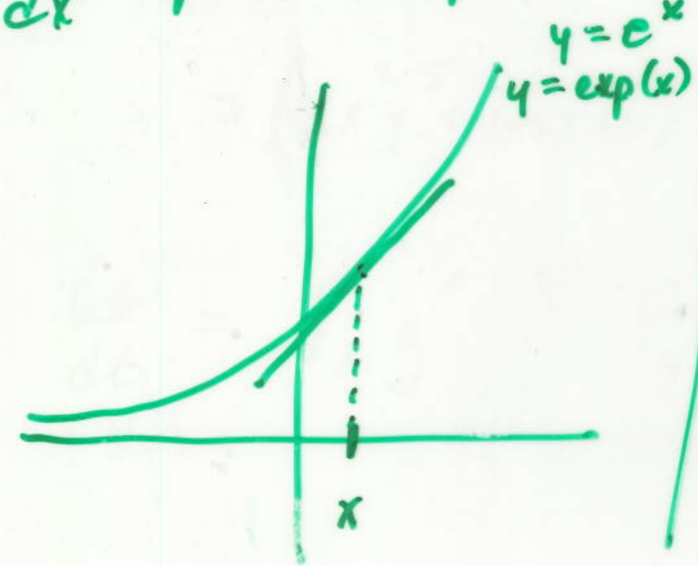
$$\frac{dy}{dx} = 1 \cdot y = y$$

$$\frac{dy}{dx} = e^x = \exp(x)$$

$$\frac{dy}{dx} = e^x = \exp(x)$$

$$\frac{d}{dx} e^x = e^x \cdot 1$$

$$\frac{d}{dx} \exp(x) = \exp(x) \cdot 1$$



Chain rule

$$\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \exp(u) = \exp(u) \cdot \frac{du}{dx}$$

$$y = 7e^{x^2+5x}$$

$$y' = 7\cdot e^{x^2+5x}\cdot(2x+5)$$

F·der S + S·der F

$$f(x) = x^2\cdot e^{3x}$$

$$f'(x) = x^2\cdot e^{3x}\cdot 3 + e^{3x}\cdot 2x$$

$$= xe^{3x}(3x+2)$$

$$\frac{d}{dx}\left(\frac{e^{x^2}}{\ln(4x)}\right) = \frac{\ln(4x)\cdot e^{x^2}\cdot 2x\cdot\frac{x}{x} - e^{x^2}\cdot\frac{1}{\cancel{4}x}\cdot\cancel{4}}{\left(\ln(4x)\right)^2}$$

$$= \frac{e^{x^2}}{x\left(\ln(4x)\right)^2}\left(\ln(4x)\,2x^2 - 1\right)$$

$$y = e^{\tan(x}$$

$$y' = e^{\tan(x^2+1)} \cdot \sec^2(x^2+1) \cdot 2x$$

$$\frac{d}{dx} \ln\left(\sin\left(e^{3x^5+2}\right)\right)$$

$$= \frac{1}{\sin\left(e^{3x^2+2}\right)} \cdot \cos\left(e^{3x^5+2}\right) \cdot e^{3x^5+2} \cdot 15x^4$$

$y = \ln u$

$dy = \frac{1}{u} \cdot du$

$\int dy = \int \frac{1}{u} \cdot du$

$y = \int \frac{1}{u} du = \int \frac{du}{u}$

$$\int \frac{du}{u} = \ln|u| + C$$

$$y = e^u$$

$$dy = e^u \cdot du$$

$$\int dy = \int e^u du$$

$$y = \int e^u du$$

$$e^u + c = \int e^u du$$

$$\boxed{\int e^u du = e^u + c}$$

and

$$\boxed{\int \frac{du}{u} = \ln|u| + c}$$

Refer to p6

$$\boxed{\int e^u\,du = e^u + C}$$

$$\frac{1}{5}\int e^{5x+2}\,5\,dx$$

let $u = 5x+2$

$du = 5\,dx$

$dx = \frac{du}{5}$ Compare

$$= \frac{1}{5}\int e^u\,du$$

$$= \frac{1}{5}e^u + C$$

$$= \frac{1}{5}e^{5x+2} + C$$

$$\boxed{\int \frac{du}{u} = \ln|u| + C}$$

$$\frac{1}{2}\int \frac{2x\,dx}{x^2+9}$$

let $u = x^2+9$

$du = 2x\,dx$

Compare

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+9| + C = \frac{1}{2}\ln(x^2+9) + C$$

$$-\frac{1}{3}\int e^{\cos(3x)}(-3)\sin 3x \, dx$$

let $u = \cos(3x)$

$du = -\sin(3x) \cdot 3\, dx$

Compare

$$= -\frac{1}{3}\int e^{u}\, du$$

$$= -\frac{1}{3} e^{u} + c = -\frac{1}{3} e^{\cos(3x)} + C$$

$$\int_0^{\pi/4} \frac{\sec^2 x \, dx}{\tan x}$$

let $u = \tan x$

$du = \sec^2 x \, dx$

Compare

$$= \int_{x=0}^{\pi/4} \frac{du}{u}$$

$$= \ln|u| \Big]_{x=0}^{\pi/4}$$

$$= \ln(\tan x) \Big]_{x=0}^{\pi/4}$$

$\ln(\tan(\pi/4)) - \ln(\tan(0))$

$\cancel{\ln 1} \rightarrow 0$ $-$ $\ln(0)$ oops, undefined

This is an improper integral.