M191 Lect #25 4-25-11

Now we define a general exponential fcn

$$a^x = e^{\ln a^x} = e^{x \ln a} \quad \text{for } a > 0$$

We define

$$a^x = e^{x \ln a} = e^{(\ln a)x}$$

$$\frac{d}{dx} a^x = \frac{d}{dx} e^{(\ln a)x} = e^{(\ln a)x} \cdot \ln a$$

$$\frac{d}{dx} a^x = a^x \ln a$$

So if $u = u(x)$

$$\boxed{\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}}$$

$$\int d\, a^u = \int a^u \ln a \, du$$

$$y = 2^x$$

$$y' = 2^x \cdot \ln 2 \doteq .7\,2^x$$

$$y = 2^{x^2+5x}$$

$$y' = 2^{x^2+5x} \cdot \ln 2 \cdot (2x+5)$$

$$y = 10^{\sin(3x)}$$

$$\frac{dy}{dx} = 10^{\sin(3x)} \cdot \ln 10 \cdot \cos(3x) \cdot 3$$

$$y = e^{5x}$$

$$y' = e^{5x} \cdot \ln e \cdot 5$$

$$\int d\,a^u = \int a^u \ln a \, du$$

$$\frac{a^u}{\ln a} + C = \frac{\int a^u \cancel{\ln} a \, du}{\cancel{\ln} a}$$

$$\boxed{\int a^u \, du = \frac{a^u}{\ln a} + C}$$

$$\frac{1}{2}\int 5^{x^2+3} \, \underbrace{2x\,dx}$$

let $u = x^2+3$

$du = 2x\,dx$

Compare

$$= \frac{1}{2}\int 5^u \, du$$

$$= \frac{1}{2}\,\frac{5^u}{\ln 5} + C = \frac{5^{x^2+3}}{2\ln 5} + C$$

General logarithm fcn

$$\boxed{y = \log_a x \quad \text{means} \quad x = a^y}$$

$$y = \log_a a^y = y \log_a a = y$$

$$x = a^{\log_a x} = x$$

We have $y = \log_a x$ means $x = a^y$

We want $\frac{dy}{dx}$

$$\frac{d}{dx} x = \frac{d}{dx} a^y$$

$$1 = a^y \cdot \ln a \cdot \frac{dy}{dx}$$

So $\frac{dy}{dx} = \frac{1}{a^y \ln a}$

$$\frac{dy}{dx} = \frac{1}{x \ln a}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

For $u = u(x)$

$$\boxed{\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \cdot \frac{du}{dx}}$$

$$d \log_a u = \frac{1}{u \ln a} \, du$$

$$\int d \log_a u = \int \frac{1}{u \ln a} du$$

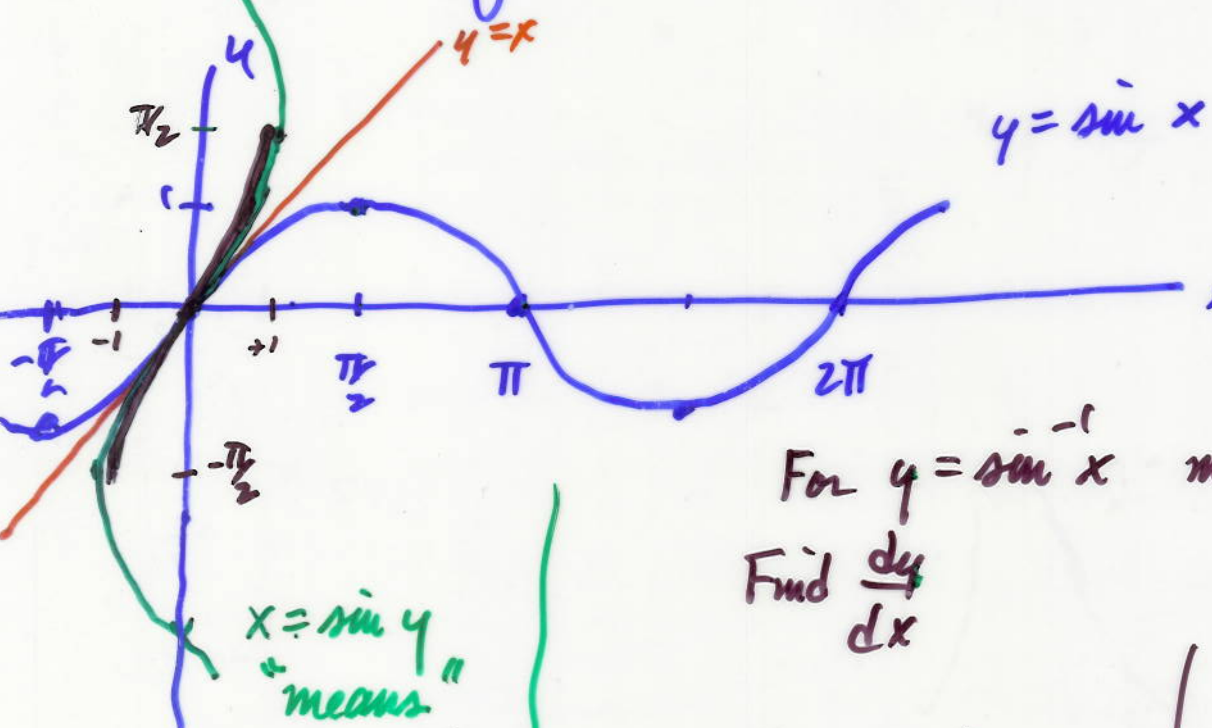
$$= \log_a u + C = \int \frac{1}{u \ln a} du$$

we actually "never" use this formula

$$y = \log_{10}(x^2+9)$$

$$y' = \frac{1}{(x^2+9)\ln 10} \cdot 2x$$

Inverse Trig Functions Review



$x = \sin y$
"means"

$y = \sin^{-1} x$

for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$-1 \le x \le 1$

For $y = \sin^{-1} x$ means $x = \sin y$

Find $\frac{dy}{dx}$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1-x^2}$

$\frac{d}{dx} x = \frac{d}{dx} \sin(y)$

$1 = \cos y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \frac{1}{\pm\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

For $u = u(x)$

$$\boxed{\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}}$$

$$d \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot du$$

$$\int d\sin^{-1}u = \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$\sin^{-1}u + C = \int \frac{du}{\sqrt{1-u^2}}$$

$$\boxed{\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}u + C}$$

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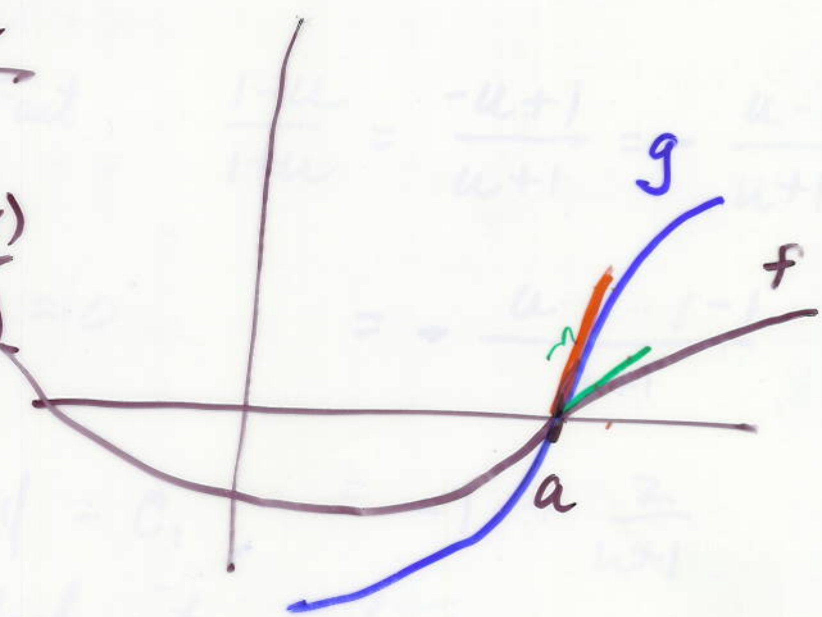
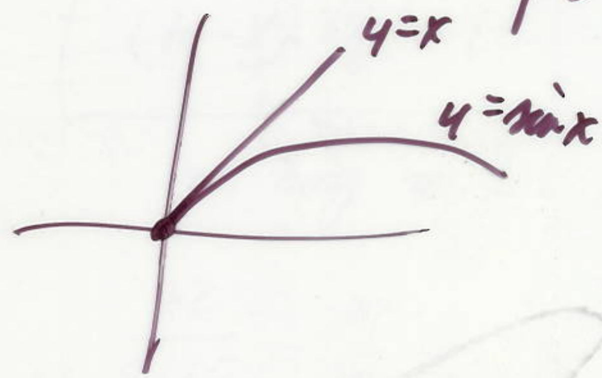
l'Hopital's Rule

$$\lim_{x\to a} \frac{\cancel{f(x)}^{\to 0}}{\cancel{g(x)}_{\to 0}} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$



this limit
is of indeterminant
form $\frac{0}{0}$

$$\lim_{x\to 0} \frac{\sin x \to 0}{x \to 0}$$



$$= \lim_{x\to 0} \frac{\cos x \to 1}{1 \to 1} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x \to 0}{x^2 \to 0}$$ I.F. $\frac{0}{0}$

$$\overset{l}{=} \lim_{x\to 0} \frac{\sin x \to 0}{2x \to 0}$$ I.F. $\frac{0}{0}$

$$\overset{l}{=} \lim_{x\to 0} \frac{\cos x \to 1}{2 \to 2} = \frac{1}{2}$$