$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\text{I.F. } \frac{\infty}{\infty}$$

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$$\text{IF } \frac{\infty}{\infty}$$

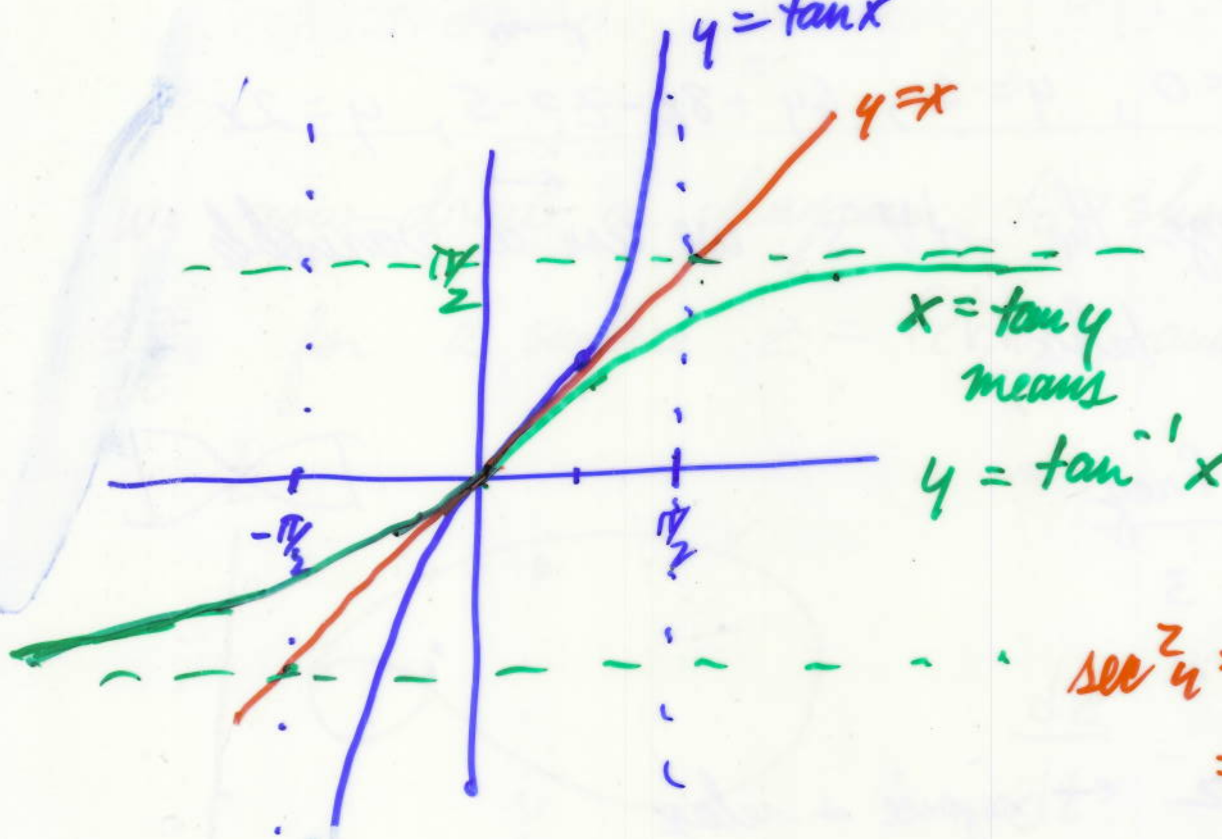
$$\lim_{x \to \infty} \frac{4^x}{x^4} = \lim_{x \to \infty} \frac{4^x \ln 4}{4x^3}$$

$$= \lim_{x \to \infty} \frac{4^x \ln 4 \cdot \ln 4 \cdot 1}{12x^2}$$

$$= \lim_{x \to \infty} \frac{4^x \cdot \ln 4 \cdot \ln 4 \cdot \ln 4}{24x}$$

$$= \lim_{x \to \infty} \frac{4^x \ln 4 \ln 4 \ln 4 \ln 4}{24} = \infty$$

Let's do the calculus for $\tan^{-1}$




$$\sec^2 y = \tan^2 y + 1$$
$$= x^2 + 1$$

For $y = \tan^{-1} x$ means $x = \tan y$

Find $\frac{dy}{dx}$

$$\frac{d}{dx} x = \frac{d}{dx} \tan y$$

$$1 = \sec^2 y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

So $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

$$\boxed{\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}}$$

$$\int d \tan^{-1} u = \int \frac{1}{1+u^2}\, du$$

$$\tan^{-1} u + C = \int \frac{du}{1+u^2}$$

$$\boxed{\int \frac{du}{1+u^2} = \tan^{-1} u + C}$$


y = cos x

$$y = \frac{(2x-5)^3 \cdot x^2}{(x+4)^6} \qquad \text{Find } \frac{dy}{dx}$$

$$\ln y = \ln \frac{(2x-5)^3 \cdot x^2}{(x+4)^6}$$

$$= + \ln (2x-5)^{3} + \ln x^{2} - \ln (x+4)^{6}$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln (2x-5) + \frac{d}{dx} 2 \ln x - \frac{d}{dx} 6 \ln (x+4)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3 \frac{2}{2x-5} + 2 \frac{1}{x} \cdot 1 - 6 \frac{1}{x+4} \cdot 1$$

$$\frac{dy}{dx} = \frac{(2x-5)^3 x^2}{(x+4)^6} \left[ \frac{6}{2x-5} + \frac{2}{x} - \frac{6}{x+4} \right]$$

## logarithmic diff.



$$y = x^x$$

prod

$$\ln y = \ln x^{x} = x \cdot \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = x^x \left[ 1 + \ln x \right]$$

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| x | y | y' |
|---|---|---|
| 2 | 5 | 8 |

$$y - 5 = 8(x - 2)$$

| t | m | m' |
|---|---|---|
| 40 | — | |
| 40 | | — |
| — | 5 | |

$$y = \log_a N$$

means

$$N = a^y$$

$$\ln N = \ln a^y = y \ln a$$

$$y = \frac{\ln N}{\ln a}.$$

$$\boxed{\log_a N = \frac{\ln N}{\ln a}}$$

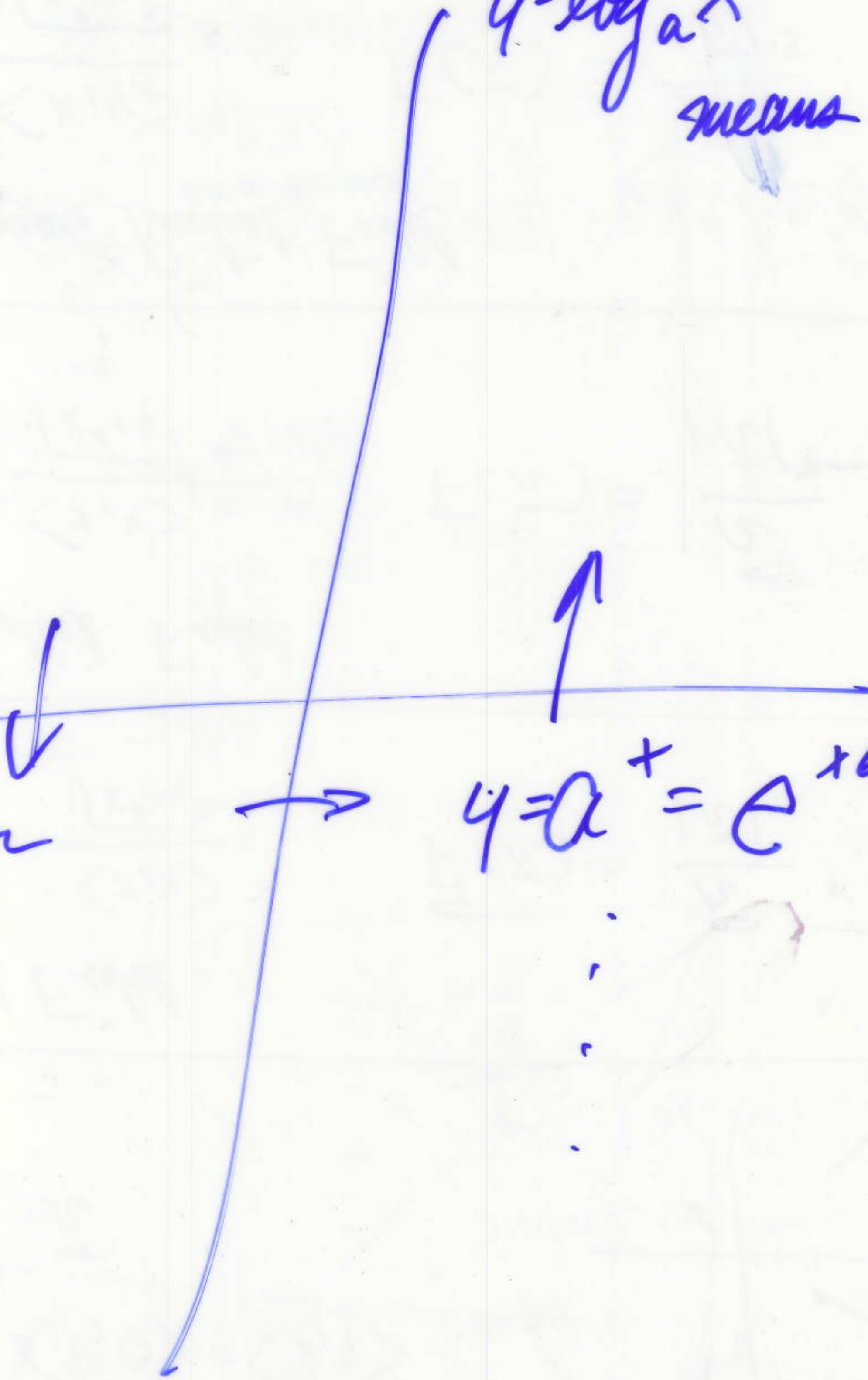
$$\frac{d}{dx} \log_a N = \frac{d}{dx} \frac{1}{\ln a} \cdot \ln N$$

$$= \frac{1}{\ln a} \frac{1}{N} \cdot \frac{dN}{dx}$$

$$\ln x = \int_1^x \frac{1}{t}\,dt$$

$y = \log_a x$

means



$y = \exp(x)$

$= e^x$

$y = a^x = e^{x \ln a}$