Math 191 Problem Set #3 for Chapter 4 C.S.Davis

Name: ________________________________

1. Most of the time in modeling "real world" situations, the mathematician doesn't have a function in closed form, but has pieces of information to synthesize into a model of the situation. Use the following puzzle of information to piece together the graph of this bizarre function.

Where there is no information in the table below, assume that the values are defined but not supplied to you. Use your best judgement to complete the graph. I recommend that you plot the given points first, determine the behavior close by those points using the t-table values in the intervals, then piece the graph together after that.

x	f(x)	f'(x)	Notes	f''(x)	Notes
-5					
(-5,-4)		**0**			
-4					f is cont @ -4
(-4,-3)				-2	
-3	2	0		-2	
(-3,-2)				-2	
-2		undef*		undef*	
(-2,-1)		**1**			
-1	4	undef*		undef*	f is cont @ -1
(-1,0)		neg		pos	
0		0		pos	
(0,1)		pos		pos	
1	-1	undef**		undef**	f is not cont @ 1
(1,2)		neg		pos	
2	4	0		0	
(2,3)		neg		neg	
3		neg		0	
(3,4)		neg		pos	
4	2	0		pos	
(4,5)		pos		pos	
5	4	undef**		undef**	f is cont @ 5
(5,6)		neg		pos	
6		neg		0	
(6,7)		neg		neg	
7	1	undef**		undef**	f is cont @ 7
(7,8)		neg		pos	
8		0		pos	
(8,9)		pos		pos	
9	undef	undef**		undef**	
(9,10)		pos		neg	
10	0				

* In these cases, the derivatives are undefined in some sense other than that their one sided limits are plus or minus infinity, i.e., the given function has a hole, a jump or a corner.

** In these cases, the derivatives are undefined in the sense that their one sided limits are plus or minus infinity, i.e., the given function has a cusp, a vertical tangent or a vertical asymptote.

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1. (Continued)

```
                     y
                  5  +
                     +
                  4  +
                     +
                  3  +
                     +
                  2  +
                     +
                  1  +
                     +
 +---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---x
-5  -4  -3  -2  -1   +   1   2   3   4   5   6   7   8   9  10
                 -1  +
                     +
                 -2  +
                     +
                 -3  +
                     +
                 -4  +
                     +
                 -5  +
                     +
```

2. Consider the function $f(x) = 3x^4 + 8x^3 - 18x^2 + 20$

a. Give the derivatives $f'(x) =$

$f''(x) =$

b. Give all critical values.

c. Give intervals where the curve is increasing and decreasing.

d. Give all hypercritical values.

e. Give intervals where the curve is concave up and concave down.

f. Give all local extrema.

g. Give all all inflection points.

1. (Continued from previous page)

h. Fill in the table below with all of the information that you have gathered along with descriptive captions like "hor. tan.", "curve rises", and "curve frowns" in appropriate places.

x	y	y'	y''

i. Now carefully graph the curve including all of the important points found above Choose a good scale and label the axes with numbers

```
                                y
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-x
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
                                +
```

3. Illustrate the Extreme Value Theorem by finding the absolute maximum and absolute minimum for the function below on the interval [-3,4].

$$f(x) = \sqrt[3]{(x-2)^2}\,(x^2 + 3x + 4)$$

a. Apply the Closed Interval Absolute Extrema Theorem to find all of the x values at which an absolute extremum could possibly occur.

b. Now give the points where the absolute maximum and the absolute minimum do occur.

4. Find the c guaranteed by the Mean Value Theorem (MVT) for the function below. Then sketch a graph of the function, the secant line and the tangent line to illustrate the MVT.

$f(x) = x^2 - 8x + 18$, defined for $2 \le x \le 5$.

5. Draw the graph of each of these functions near the value x = 3. Then give a term (word or phrase) which describes the behavior of the graphs of each function at and near the value of x=3. Then

a. $f(x) = (x - 3)^{2/3} + 2$ ____________________________________

```
            y
            +
            +
            +
            +
            +
            +
+-+-+-+-+-+-+-+-+-+-+-+-x
            +
            +
            +
            +
```

b. $f(x) = (x - 3)^{1/5} + 2$ ____________________________________

```
            y
            +
            +
            +
            +
            +
            +
+-+-+-+-+-+-+-+-+-+-+-+-x
            +
            +
            +
            +
```

c. $f(x) = (x - 3)^{-1} + 2$ ____________________________________

```
            y
            +
            +
            +
            +
            +
            +
+-+-+-+-+-+-+-+-+-+-+-+-x
            +
            +
            +
            +
```

6. Suppose the vertical position s of an object is given by the formula,

$$s = -2t^3 + 15t^2 - 24t + 20, \quad \text{for } t \geq 0 \text{ and } s \geq 0.$$

a. Find the time when the object reaches its maximum height.

b. Find the maximum height.

c. Find the time when the velocity is greatest.

d. Find the greatest velocity.

e. Find the greatest acceleration of the object.

f. When does the object hit the ground. (Use Newton's method on $f(t) = 0$, show the iteration formula, and give the answer to twelve significant figures.)

For each of the optimization problems below, include an objective function, constraints and identification of what you are to find. Write the answer in a sentence. Use some theorem or test to assure that your answer is optimal. Please begin each problem on a separate sheet of paper.

7. The sides of a rectangular picture frame cost \$6 per foot and the top and bottom cost \$3 per foot. Find the dimensions that will maximize the area that the picture frame encloses (picture and frame) for a total cost of \$60. (We want the largest coverage of the wall where the picture will be hung.)

8. A power station and a resort are on opposite sides of a river 2 thousand feet across. The resort is 5 thousand feet down stream from the power station. If it costs 8 dollars per foot to run power lines along the side of the river and 12 dollars per foot to run the power under the river, find the combination path through the river and along the side that will cost the least.

9. A manufacturer wishes to produce a can of volume 12π cubic inches as cheaply as possible. The tin for the top and bottom costs 3 cents per square inch and the tin for the side costs 2 cents per square inch. Find the dimensions (radius and height) of the can whose cost for production is a minimum.

10. Name and address each of the first 4 steps (A - D) of Stewart's 8 steps of graphing for the function below. (We have already done steps E - H in problem 2.) Then graph the function identifying any of the four aspects in the graph: domain, intercepts, symmetry and vertical and horizontal asymptotes.

$$f(x) = \frac{-x^3 + x}{x^2 + x - 6}$$