1. Given the function $f(x) = \frac{1}{x-4}$

   a. Determine whether f is one-to-one. Give a reason.

   b. Find f's inverse function and call it g. Give g(x).

   c. Graph f and g on the same set of axes.

   d. Give the domain and range for f and for g.

   e. Find the derivative of f and g.

   f. Note that f(6) = ½. Use the values of 6 and ½ to demonstrate the connection between f' and g'.

   g. Give the equation of the tangent line to y = f(x) @ x=6 and the equation of the tangent line to y = g(x) @ x=1/2.

h. Explain how this (seemingly false) statement can be interpreted to be true:

"The derivative of the inverse is the inverse of the derivative."

2. Find the derivative of each of the following functions.

   a. $f(x) = \ln (x^3 (7x-4)^{2)})$

   b. $f(x) = \ln (\csc^2(5x^2))$

3. Differentiate implicitly to find dy/dx.

   $\ln (x^2 y^3) + x^4 y^5 = 12$

4. Use logarithmic differentiation to find the derivative of g.

   $g(x) = (x^2 + 5)^4 (x^3 - 8)^5 \sqrt{x^4 + 17}$

5. Evaluate these integrals.

a. $\displaystyle\int \frac{(e^{x} + 3)^2}{e^{x}}\, dx$

b. $\displaystyle\int \frac{x^2 + 4}{x^3 + 12x + 5}\, dx$

c. $\displaystyle\int \frac{e^{x}}{(e^{x} + 12)^3}\, dx$

d. $\displaystyle\int (\tan 13x + \sec 8x)\, dx$

6. Find the equation of the tangent line to the curve $y = 5^x$ at $x=2$.

7. Find the exact area of the region bounded above by the curve $y = 3/x$, below by the curve $y = e^{-x}$, on the left by the line $x=1$ and on the right by the line $x = 4$. Draw a graph of the region.

8. Integrate $\int 2^{\sin(3x)} \cos(3x)\, dx$

9. If the initial mass of a sample of ^{90}Sr is 10mg, the the mass after t years is given by

$m(t) = 10\,e^{-(\ln 2)\,t/25}$.

a. Find the mass remaining after 40 years.

b. At what rate does the mass decay after 40 years?

c. How long does it take for the mass to be reduced to 5mg? (This is the half-life of Strontium 90, a deadly radioactive isotope of strontium occurring in the fallout of nuclear explosions)

10. Find the derivatives of the following functions.

a. $f(x) = (2x)^{(3x)}$

b. $f(x) = \log_7 (x^2 + 1)$

11. Give and overview of the development of logarithms and exponentials. Define each of the four functions, showing how each is derived from the ones before. Give graphs and major properties. Write small and fill up the page.

12. For the inverse cosine function, $\cos^{-1}$, do all of the following.

a. Give the definition, domain & range.

b. Evaluate $\cos^{-1}(-½)$

c. Draw the graph.

d. Find the derivative of $\cos^{-1}(3x)$.

e. Evaluate this integral.

$$\int_0^{1/2} -\frac{1}{\sqrt{9-x^2}}\,dx$$

f. Derive the formula for the derivative of $y = \cos^{-1}(x)$.

13. For the hyperbolic secant function, sech, do the following.

a. Give the definition, domain & range.

b. Evaluate sech (½)

c. Draw the graph.

d. Find the derivative of sech (3x).

e. Evaluate this integral.

$$\int_0^2 sech(3x)\ \tanh(3x)\ dx$$

f. Derive the formula for the derivative of y = sech (x).

14. Determine whether l'Hopital's rule applies to the following and find the limit in each.

a. $\lim_{x \to 0} \frac{x + \tan x}{\sin x}$

b. $\lim_{x \to \infty} \frac{e^x}{x^4}$

c. $\lim_{x \to \infty} \frac{e^x}{x^n}$, where n is a fixed positive integer constant.

Draw a conclusion about the growth rate of exponential functions versus power functions.

d. $\lim_{x \to 0^+} x^2 \ln x$

e. $\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

f. $\lim_{x \to 0^+} (\sin x)^{(\tan x)}$

g. $\lim_{x \to 0^+} (csc\ x)^{(\tan x)}$

h. $\lim_{x \to 0^+} (\cos x)^{\left(\frac{5}{x}\right)}$