Math 192 Lecture #1

18+15-1

While I take roll, would you do the following.

For the function $y = f(x) = 2x^2 + 5x - 1$

x	y	y'
2	17	13

a) Find the slope of the tangent line at $x=2$.

$y' = f'(x) = 4x + 5$

1 6

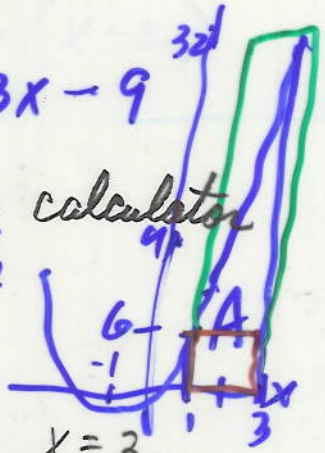
b) Find the equation of the tangent line at $x=2$. 3

$y - y_1 = m(x - x_1)$

$y - 17 = 13(x - 2)$ or $y = 13x - 9$

c) Graph the function f and its tan line on the calculator

$y =$ $2x^2+5x-1$ $13x-9$ graph



d) Estimate the area under f between $x=1$ and $x=3$.

(2)6 ≤ A ≤ 32(2)

12 ≤ A ≤ 64

$A \approx \frac{64+12}{2} = 38$

e) Find the exact area using calculus (FTC 2)

$$A = \int_1^3 (2x^2+5x-1)\,dx = 2\frac{x^3}{3} + \frac{5x^2}{2} - x \Big]_1^3$$

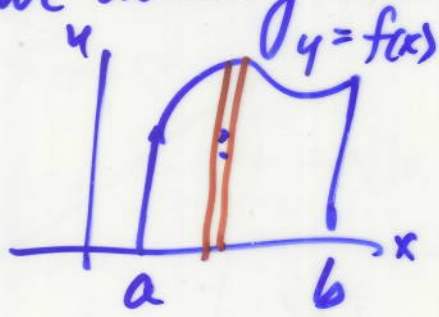
$$= \frac{2(27-1)}{3} + \frac{5(9-1)}{2} - (3-1)$$

$$= \frac{52}{3} + 20 - 2 = 17\tfrac{1}{3} + 18 = 35\tfrac{1}{3}$$

M192 Lect #1 8-22-11

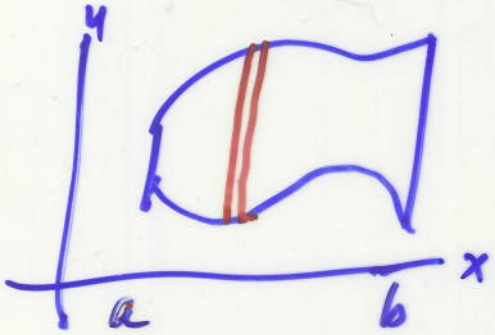
§6.1 Areas

We have already done areas that look like this

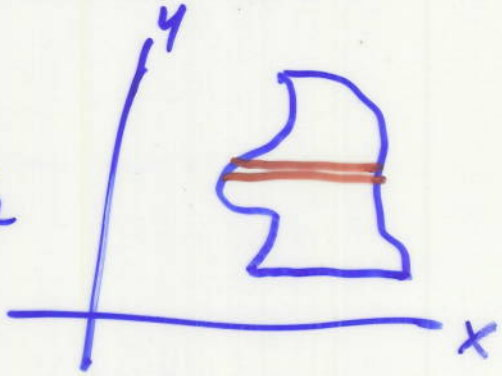


$$A = \int_{x=a}^{b} \underbrace{f(x)\,dx}_{dA}$$

We now learn how to find area like this



and this



Here is the overriding concept

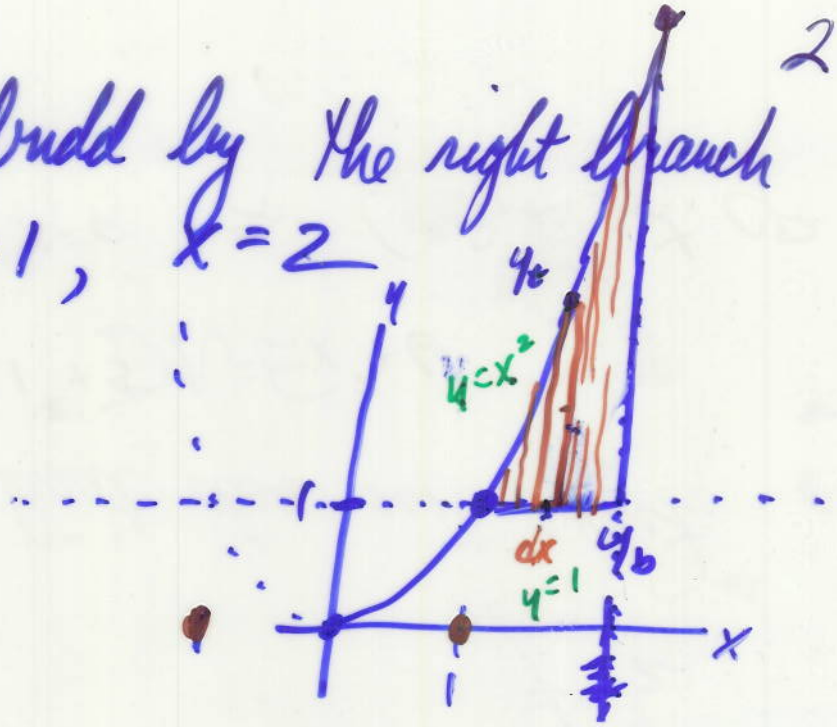
The whole is the sum of its parts

$$A = \int_{a}^{A} dA = \int dA \Big]_{0}^{A} = A \Big]_{0}^{A} = A - 0 = A$$

$$V = \int_{0}^{V} dV$$

Ex Find the area bndd by the right branch of $y = x^2$, $y = 1$, $x = 2$

y_t $\quad$ y_b



$$A = \int_0^A dA = \int_{x=1}^{2} (y_t - y_b)\,dx = \int_1^2 (x^2 - 1)\,dx$$

$$= \frac{x^3}{3} - x \Big]_1^2 = \frac{8-1}{3} - (2-1)$$

$$= 7/3 - 1 = \frac{4}{3} = 1\tfrac{1}{3} \doteq 1.33$$

$y_t = x^2$

$y_b = 1$

$y_t \overset{\text{set}}{=} y_b$

$x^2 = 1$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = \pm 1$

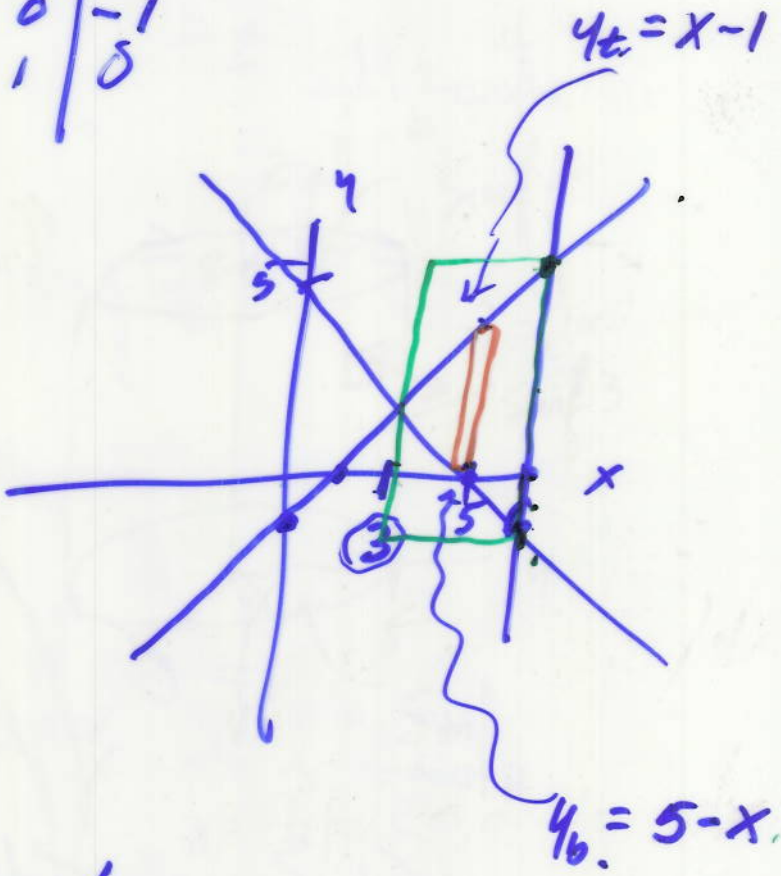
$x = +1$ only on applic.

Ex Find A of region bndd by

$x+y=5$ $\quad x-y=1$ $\quad x=6$

x	y
0	5
5	0

x	y
0	-1
1	0



$$A = \int_0^A dA$$

$$= \int_3^6 (y_t - y_b)\,dx$$

$$= \int_3^6 (x-1-(5-x))\,dx$$

$$= \int_3^6 (2x-6)\,dx = x^2-6x\Big]_3^6$$

$$= 36-36-(9-18)$$

$$= 9$$

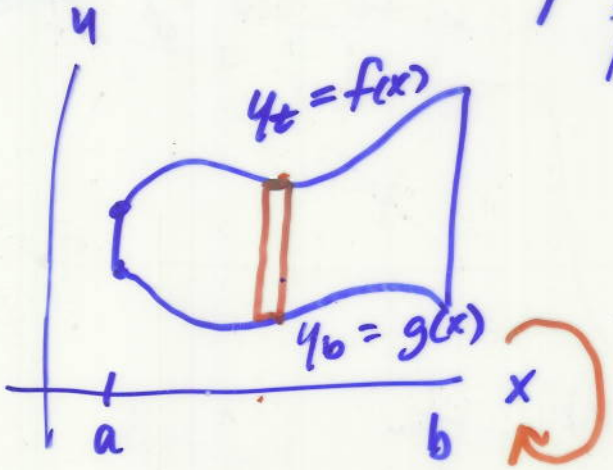
$y_t \overset{\text{set}}{=} y_b$

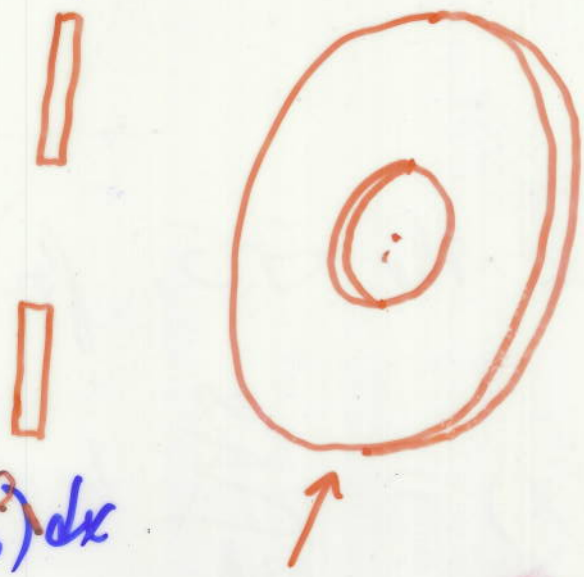
$x-1 = 5-x$

$2x = 6$

$x = 3$

§6.2 Volumes of Solids of Revolution: Method of Disk (Washers)





$$dA = (y_t - y_b)\,dx$$

$$A = \int_0^A dA$$

$$V = \int_0^V dV$$

$$= \int_a^b \pi(y_t^2 - y_b^2)\,dx$$

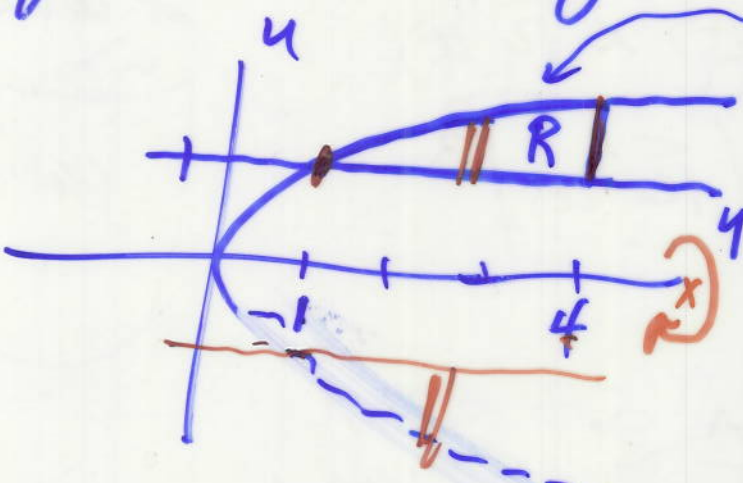
$$dV = (\pi R_o^2 - \pi R_i^2)\,dx$$

$$dV = (\pi y_t^2 - \pi y_b^2)\,dx$$

$$dV = \pi(y_t^2 - y_b^2)\,dx$$

Find the vol of the solid of rev generated by revolving this region about the x-axis.

Region is bndd by $x=y^2$, $y=1$, $x=4$



$x=y^2$ or $y_t=+\sqrt{x}$

$dA=(y_t-y_b)dx$

$dV=\pi(y_t^2-y_b^2)dx$

$$V=\int_0^V dV=\int_1^4 \pi(y_t^2-y_b^2)dx$$

$$=\int_1^4 \pi(\sqrt{x}^2-1^2)dx \quad =\pi\int_1^4 (x-1)dx$$

$$=\pi\left[\frac{x^2}{2}-x\right]_1^4 \quad =\pi\left[\frac{16-1}{2}-(4-1)\right]$$

$$=\pi\left[\frac{15}{2}-3\right]=\left[7\tfrac{1}{2}-3\right]\pi=4\tfrac{1}{2}\pi=4.5\pi\approx 14$$