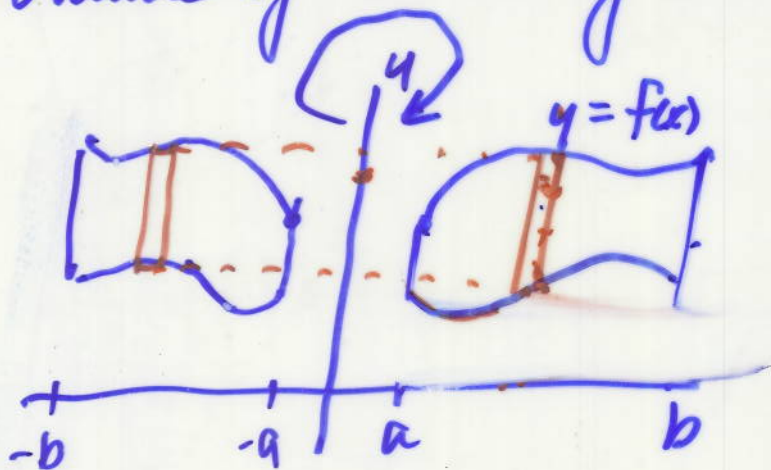


M192

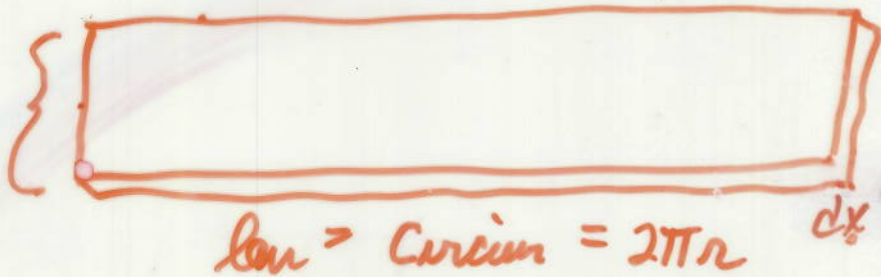
Lect 2

8-24-11

§ 6.3 [?] Method of Shells for finding
volume of solids of revolution



$$ht = y_t - y_b$$



$$dV = l \cdot h \cdot w$$

$$= 2\pi x$$

$$dV = \underbrace{2\pi x}_{\text{Circum}} (y_t - y_b) dx$$

$$\text{But } dA = (y_t - y_b) dx$$

$$V = \int_0^V dV = \int_a^b 2\pi x (y_t - y_b) dx$$

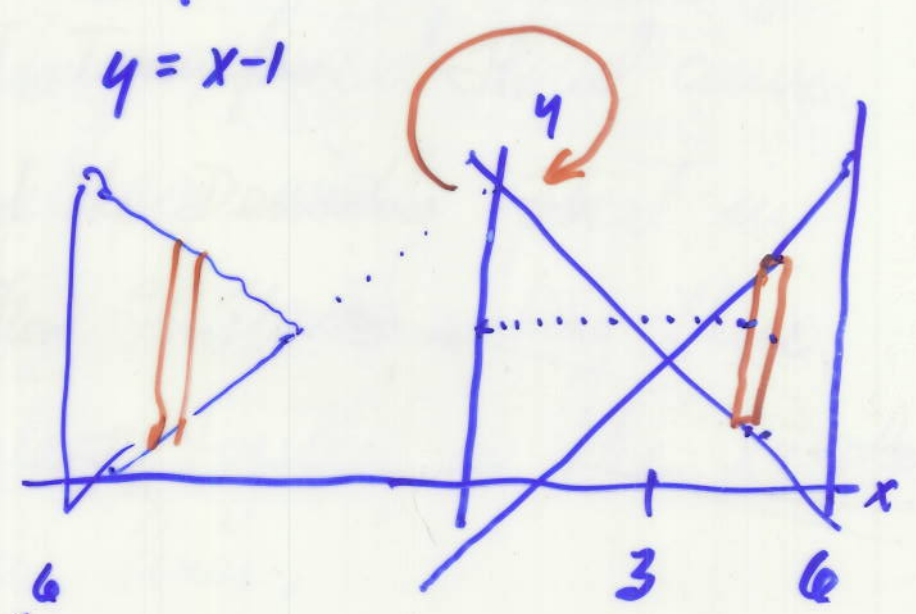
Ex Find the volume of the solid of rev
 generated by revolving this region
 about the _{bot} y -axis. Use Shells

$x+y=5$
 $y=5-x$

^{top}
 $x-y=1$
 $y=x-1$

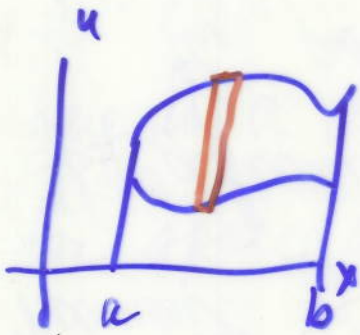
$x=6$

$dA = (y_t - y_b) dx$



$$\begin{aligned}
 V &= \int_0^6 dV = \int_3^6 2\pi x (y_t - y_b) dx \\
 &= \int_3^6 2\pi x (x-1 - (5-x)) dx = 2\pi \int_3^6 (2x^2 - 6x) dx \\
 &= 2\pi \left[2 \cdot \frac{x^3}{3} - \frac{6}{2} \frac{x^2}{2} \right]_3^6 = 2\pi \left[\frac{2}{3} (216 - 27) - 3(36 - 9) \right] \\
 &= 2\pi [126 - 81] = 2\pi(45) = 90\pi \doteq 282 \mu\text{AO}
 \end{aligned}$$

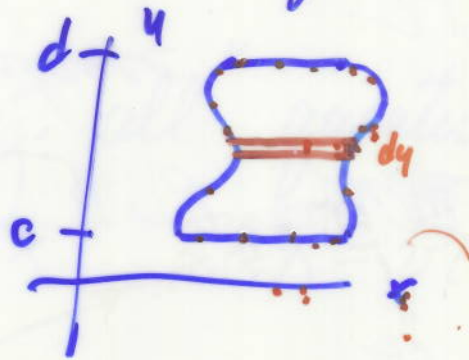
For Vert Rect



$$dA = (y_t - y_b) dx$$

$$A = \int_0^A dA = \int_a^b (y_t - y_b) dx$$

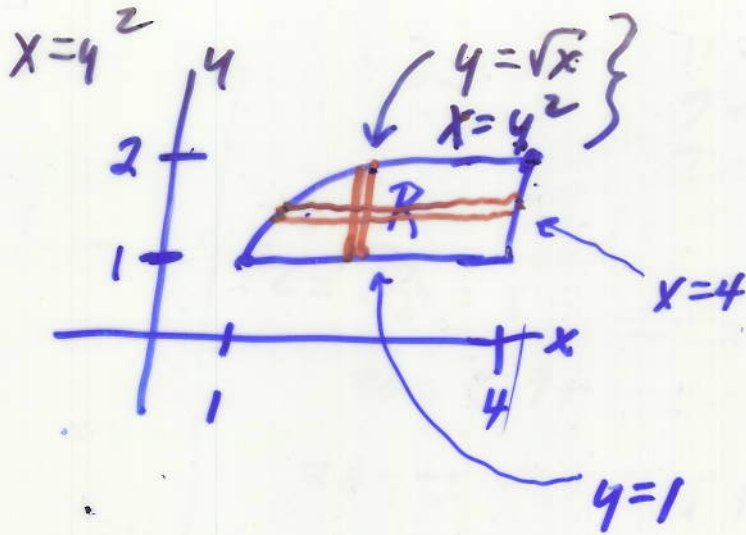
For horizontal Rect









$$dA = (x_t - x_b) dy$$

$$A = \int_0^A dA = \int_c^d (x_t - x_b) dy$$

For the region



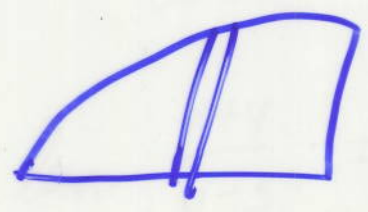
Find

- ① A of Reg, use vert rect , dx
- ② A of Reg, use hor rect , dy
- ③ V of Solid, rev Reg about x axis, use vert rect  dx method of disks
- ④ V of Solid, rev Reg about x axis, use hor rect  dy method of shells
- ⑤ V of Solid, rev R about y axis, use vert rect  dx method of shells
- ⑥ V of Solid, rev R about y axis, use hor rect , dy method of disks

$$\textcircled{1} A = \int_0^A dA = \int_1^4 (y_+ - y_-) dx = \int_1^4 (\sqrt{x} - 1) dx$$

⋮

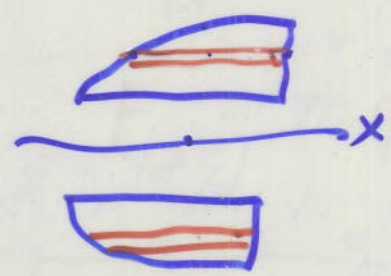
$$= 1.6\bar{6} = \frac{5}{3}$$



$$\textcircled{3} V = \int_0^V dV = \int_{1/2}^4 \pi \left(\frac{y_+}{2} - \frac{y_-}{2} \right)^2 dx$$

$$\textcircled{2} A = \int_0^A dA = \int_1^4 (\sqrt{x} - (x - x^2)) dy = \int_1^4 (4 - y^2) dy$$

$$= \frac{14.14}{1.6\bar{6}} = \frac{5}{3}$$



$$\textcircled{4} V = \int_0^V dV = \int_c^d 2\pi y (x_+ - x_-) dy$$

$$= \int_1^2 2\pi y (4 - y^2) dy$$

⑤ $V = \int_0^v dV = \int_a^b 2\pi x (y_t - y_b) dx$

$= \int_1^4 2\pi x (\sqrt{x} - 1) dx = 2\pi \int_1^4 (x^{3/2} - x) dx$

$= 2\pi \left[\frac{x^{5/2}}{5/2} - \frac{x^2}{2} \right]_1^4$

$= 30.79$

⑥ $V = \int_0^v dV = \int_c^d \pi (x_t - x_b) dy$

$= \int_1^2 \pi (4^2 - (y^2)^2) dy$

\vdots

$= 30.79$