M192 Lect #3 8-31-11

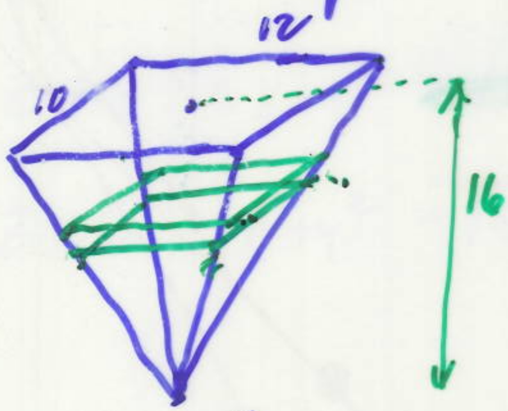
Tonight Method of Slices

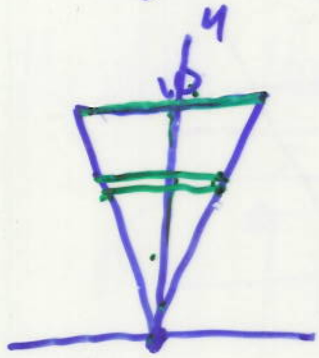
Work < spring, pumping, dangling cable

Ex. Method of Slicing

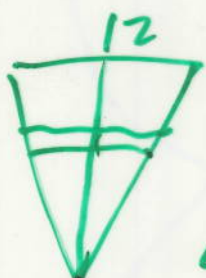
Prin: the whole is the sum of its parts

$V = \int_0^V dV$





$\frac{y}{16} = \frac{w}{10}$ $\quad w = \frac{10y}{16}$



$\frac{y}{16} = \frac{l}{12}$ $\quad l = \frac{12y}{16}$

$dV = l \cdot w \cdot ht = \frac{12y}{16} \cdot \frac{10y}{16} \cdot dy$

$V = \int_0^V dV = \int_0^{16} \frac{12 \cdot 10}{16 \cdot 16} y^2 \, dy$

Spring Problem Find the Work done in stretching a spring.

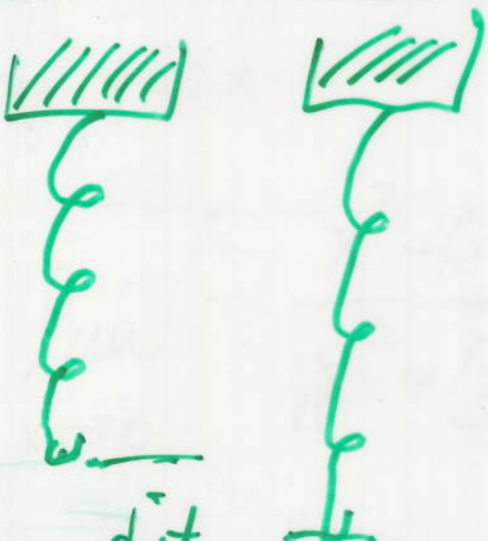
Principle: The whole work is the sum of the little bitty works.

$$W = \int_0^W dW$$

Work = force · distance

$$dW = F \cdot dx = x \cdot dF$$

most often used



Hooke's Law

The distance x a spring stretches is proportional to the force acting on it.

$$F = kx$$

Here is a typical condition to find k.

A 5 # force stretches a spring 3 feet

$5 = k \cdot 3$

So $k = \frac{5}{3}$

Now

$$F = \frac{5}{3}x$$

$$W = \int_0^W dW = \int_a^b F \cdot dx = \int_a^b \frac{5}{3} x\,dx$$

A spring 50 feet long is stretched 3 feet by a force of 5 lbs. How much work is done in stretching the spring from 2 feet beyond its natural length to 6 feet beyond its natural length

$W = \int_0^{W_0} dW \qquad dW = F \cdot dx \qquad F = kx$

$5 = k\,3$

$k = \frac{5}{3}$

$F = \frac{5}{3}x$

$W = \int_0^W dW$

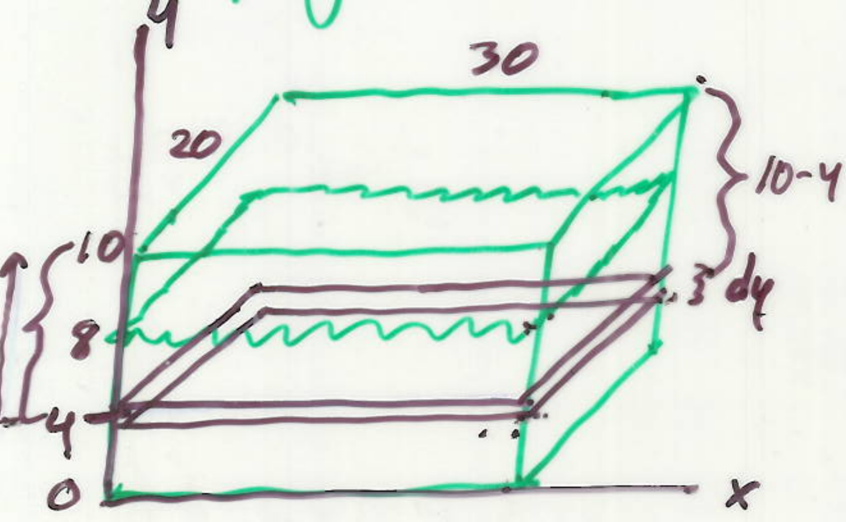
$= \int_2^6 F \cdot dx = \int_2^6 \frac{5}{3} x \, dx$

$= \frac{5}{3} \frac{x^2}{2} \Big]_2^6 = \frac{5}{3}(18 - 2) = \frac{80}{3} = 26\frac{2}{3}$

ft #

ft lbs
of work

Pumping Problem (a Work problem)



$$W = \int_0^W dW$$

density of water is
$62.5 \ \#/ft^3$

$$\dot{W} = \int_0^w dw$$

$$= \int_0^8 F \cdot dx$$

$$= \int_0^8 600 \times 62.5(10-y)\,dy$$

$\vdots$

$= 1.8$ million ft# of work

$dV = \text{len} \cdot \text{wid} \cdot \text{depth}$

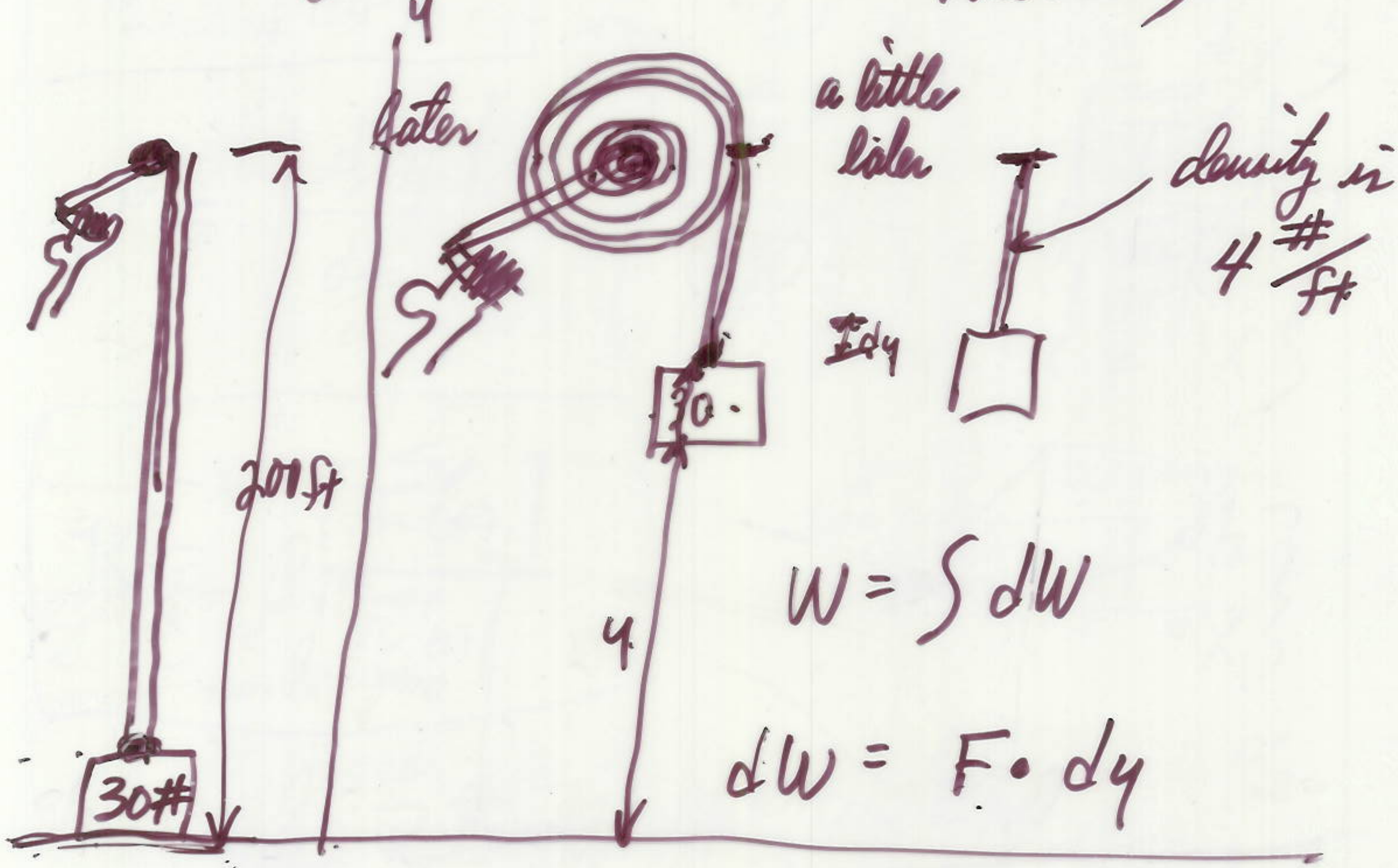
$dV = 30 \cdot 20 \cdot dy$

$F = \text{den} \cdot \text{vol} = 62.5 \cdot 30 \cdot 20\,dy$

$dW = F \cdot \text{dist}$

$dW = 62.5(30)(20)\,dy \cdot (10-y)$

Dangling Cable Problem (Another Work Problem)



$$W = \int dW$$

$$dW = F \cdot dy$$

$$F = \left(30 + (200 - y)\cdot 4\right)$$

Work from bottom to top

$$W = \int_0^W dW = \int_{y=0}^{200} \left(30 + 4(200 - y)\right) dy$$

Work from 1/4 mark to the 3/4 mark

$$W = \int_{50}^{150} \left(30 + 4(200 - y)\right) dy$$