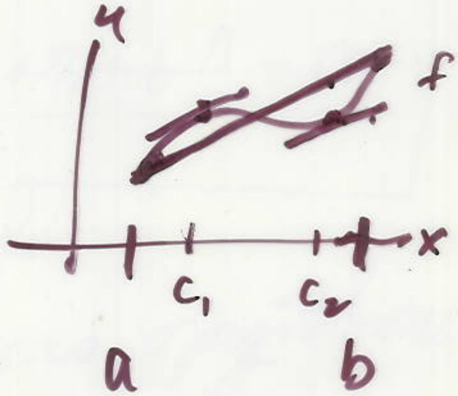
M 192 Lect #4 9-7-11

§6.5 Mean Value Theorem for Integrals

The old MVT in Calc I looked like this

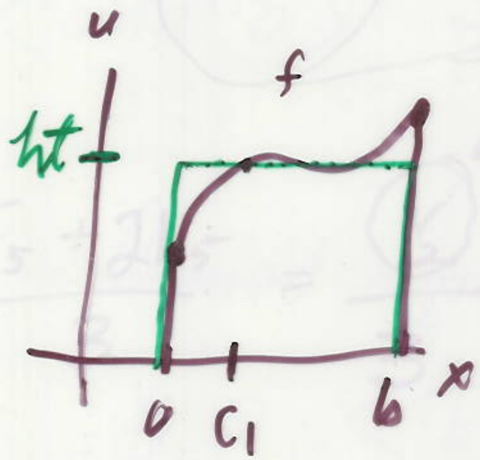


Let f be cont & diff on [a,b]

$$\frac{f(b)-f(a)}{b-a} = f'(c_1) = f'(c_2)$$

The is an x-value C_1 such that the above is true.

MVT for Int in Calc II looks like this



$$ht \cdot (b-a) = \int_a^b f(x)\,dx$$

If f is cont on [a,b]

There is an x-value C_1 $a \le c_1 \le b$ such that

$$f(c_1) \cdot (b-a) = \int_a^b f(x)\,dx$$

$f(c_1)$ is called the mean value of the function on [a,b]

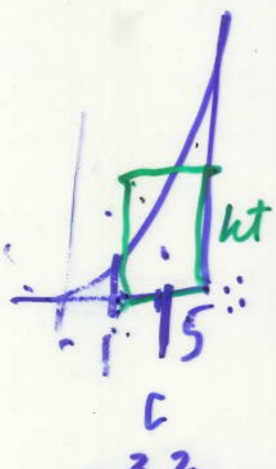
Ex For the fcn $f(x) = x^2+2$ defined on $[1,5]$

① find the mean value (ht) of f on $[1,5]$

② find the x value $(c,)$ where the mean value occurs.

---

① $\int_a^b f(x)\,dx = \int_1^5 (x^2+2)\,dx = \frac{x^3}{3}+2x\Big]_1^5$



$$= \frac{125-1}{3} + 2(5-1) = \frac{124}{3} + 8\cdot\frac{3}{3}$$

$$= \frac{124+24}{3} = \frac{148}{3} = 49\frac{1}{3}$$

$$\frac{\frac{148}{3}}{\frac{5-1}{1}} = \frac{148}{3}\cdot\frac{1}{4} = \frac{37}{3} = 12\frac{1}{3} = ht$$

= the mean value of f over $[1,5]$

---

② $f(x) = x^2+2 \overset{set}{=} \frac{37}{3}$

$f(c) = c^2+2 = \frac{37}{3}$

↳

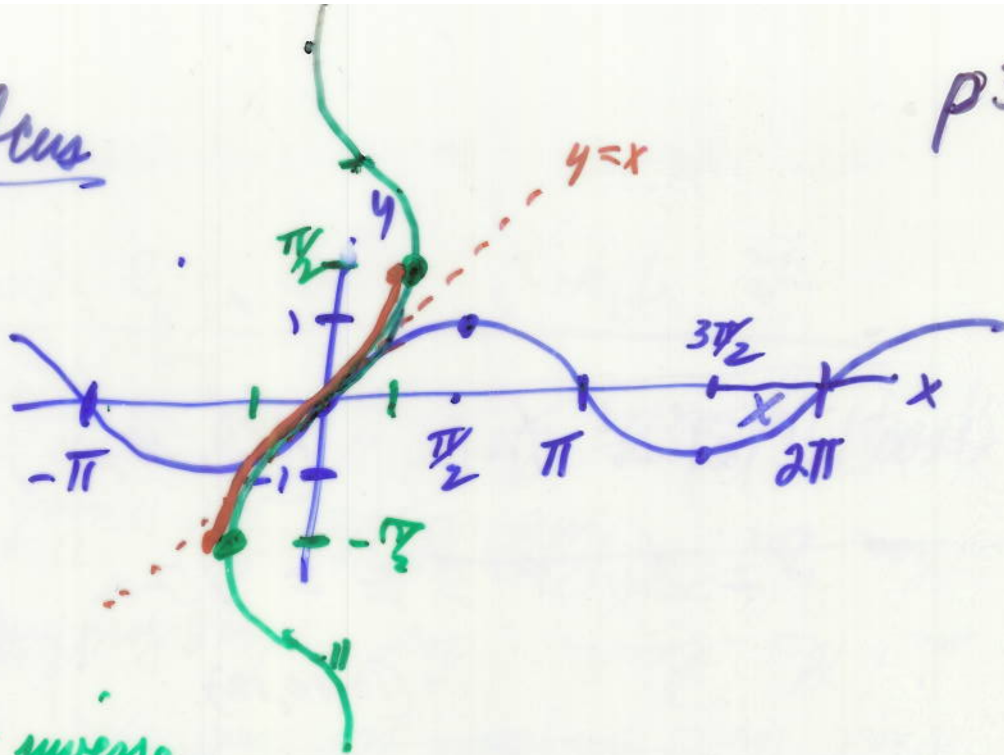
$c^2 = \frac{37}{3} - \frac{6}{3} = \frac{31}{3}$

$c = \pm\sqrt{\frac{31}{3}}$ or $c = +\sqrt{\frac{31}{3}}$

$c \approx 3.2$

§7.6 Inverse trig fcns

$y = \sin x$



$x = \sin y$ ← inverse relation

means

the red bite

$\boxed{y = \sin^{-1} x}$ for those value of y between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$

by the way $-1 \le x \le 1$

To find the derivative of $y = f(x) = \sin^{-1} x$

i.e. to find $y' = \frac{dy}{dx}$, we write what it "means", diff that & solve for $y' = \frac{dy}{dx}$

$y = \sin^{-1} x$ means $x = \sin y$.

$$\frac{dx}{dx} = \frac{d \sin y}{dx}$$

$$1 = \cos y \cdot y'$$

$$y' = \frac{1}{\cos y}$$

Recall $\sin^2 y + \cos^2 y = 1$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \pm\sqrt{1-\sin^2 y}$$

$$= \pm\sqrt{1-x^2}$$

$$= \pm\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

Now the chain rule version is

$$\boxed{\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}}$$

$$\boxed{\int \frac{1}{\sqrt{1-u^2}}\, du = \sin^{-1} u + C}$$

l'Hopital's rule

$$\lim_{x \to 0} \frac{\overset{\to 0}{\sin x}}{\underset{\to 0}{x}} = \lim_{x \to 0} \frac{\overset{\to 1}{\cos x}}{\underset{\to 1}{1}} = 1$$