M192 Lect #5 9-12-11

## l'Hopital's rule

I.F. Indeterminant Forms

In Calc I

$$\lim_{x \to 2} \frac{x^2-4}{x-2} \qquad \text{IF } \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = 4$$

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l'Hopital says do this

$$\lim_{x \to 2} \frac{x^2-4}{x-2} \overset{H}{=} \lim_{x \to 2} \frac{2x}{1} = 4$$

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If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to 0} g(x) = 0$

& f, g are cont & diff at a.

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

IF $\frac{0}{0}$ p2

$$\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$y = \sin^2 x$$

$$y = (\sin x)^2$$

$$2\sin x \cdot \cos x$$

$$= \lim_{x\to 0} \frac{\cancel{\sin x}}{2\cancel{\sin x} \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{1}{2\cos x} \quad \begin{matrix}\to 1\\ \to 2\end{matrix} = \frac{1}{2}$$

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$$\lim_{x\to 0} \frac{x \sin x}{1-\cos x} \quad \begin{matrix}\to 0\\ \to 0\end{matrix} \overset{H}{=} \lim_{x\to 0} \frac{x \cdot \cos x + \sin x \cdot 1}{0 + \sin x} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$= \lim_{x\to 0} \frac{-x \cdot \sin x + \cos x \cdot 1 + \cos x}{\cos x} \quad \begin{matrix}\to 2\\ \to 1\end{matrix} = \frac{2}{1} = 2$$

$2^{nd}$ I.F. is $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{5^x}{x^5} \;(\text{I.F.}\ \tfrac{\infty}{\infty}) = \lim_{x\to\infty} \frac{5^x \cdot \ln 5}{5x^4}$$



The last two were called I.Q. quotients

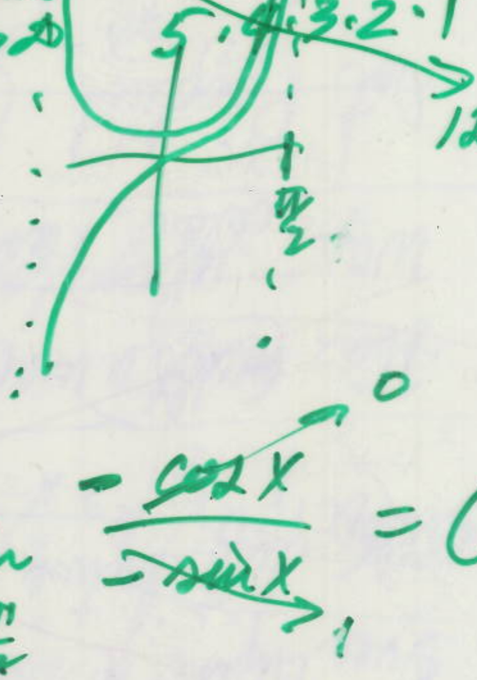
$$= \lim_{x\to\infty} \frac{5^x \cdot \ln 5 \cdot \ln 5}{5\cdot 4\, x^3} = \lim_{x\to\infty} \frac{5^x (\ln 5)^3}{5\cdot 4\cdot 3\, x^2}$$

(3) is I.D. diff $\to \infty$

$$= \lim_{x\to\frac{\pi}{2}} \sec x - \tan x$$

$$\lim_{x\to\infty} \frac{5^x (\ln 5)^4}{5\cdot 4\cdot 3\cdot 2\, x}$$

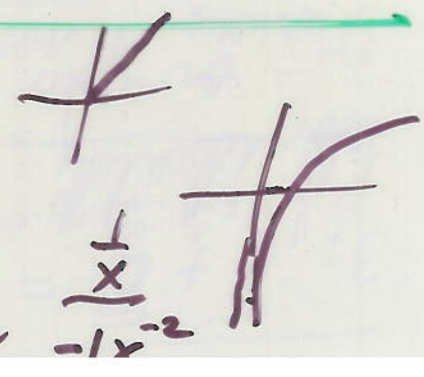
I.F.: $\infty$ $$= \lim_{x\to\infty} \frac{5^x (\ln 5)^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} \quad (\to 120)$$



$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{H}{=} \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0$$

(4) is I Prod : $0\cdot\infty$ ← product



$$\lim x\cdot \ln x \;\; (0\cdot(-\infty)) = \lim \frac{\ln x}{x^{-1}} = \lim \frac{\frac{1}{x}}{-1x^{-2}}$$

The last three are I Pow ← powers

$1^{\infty}$, $\infty^{0}$, $0^{0}$

$e^{\ln x} = x$ ← p6

$\ln e^{x} = x$

Chapter 8 Integration Techniques

Of I Pow

$\lim x^{x}$

$\int x \cdot \cancel{\ln x}$ + Integration by Parts

(In limits - Product Rule)

$= \lim_{x \to 0^+} e^{\ln x^x}$

$d(u \cdot v) = u \cdot dv + v \cdot du$

$= e^{\lim_{x \to 0^+} \ln x^x}$

$\int d(u \cdot v) = \int u \cdot dv + \int v\, du$

$= e^{\lim_{x \to 0^+} x \ln x}$

$u \cdot v$ IQ $\frac{\infty}{\infty}$

$= e^{\lim_{x \to 0^+} \frac{\ln x}{x^{-1}}}$

$uv - \int v\, du = \int u\, dv$

$= e^{\lim_{x \to 0^+} \frac{\frac{1}{x}}{-x^{-2}}}$

$= e^{\lim_{x \to 0^+} -x} = e^{0} = 1$

$$\boxed{\int u\, dv = u \cdot v - \int v\, du}$$

<u>Ex</u> $\int u\,dv = u\cdot v - \int v\,du$

$\int x e^x \, dx$

$= x\cdot e^x - \int e^x \, dx$

$u = x \qquad dv = e^x dx$

$du = 1\,dx \qquad v = e^x$

$= xe^x - e^x + c$

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Check

$\frac{d}{dx}\left(xe^x - e^x + c\right) = xe^x + e^x \cdot 1 - e^x + 0$

$= xe^x$