M 192 Lect #6 9-14-11

Ex Integrate by parts

$\int u\,dv = uv - \int v\,du$

$\int x^2 e^{3x}\,dx$

$u = e^{3x}$ $\quad dv = x^2\,dx$

$du = 3e^{3x}$ $\quad v = \frac{x^3}{3}$

$= e^{3x}\frac{x^3}{3} - \int \frac{x^3}{3} 3e^{3x}\,dx$ ← look worse than original

Bad choice

Try another sub

$\int x^2 e^{3x}\,dx$

$u = x^2$ $\quad dv = \frac{1}{3}e^{3x}3\,dx$

$du = 2x\,dx$ $\quad v = \frac{1}{3}e^{3x}$

$= x^2 \frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x} 2x\,dx$

$= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x e^{3x}\,dx$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}\,dx$$

$u = x$ $\quad dv = e^{3x}\,dx$

$du = dx$ $\quad v = \frac{1}{3}e^{3x}$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[x\,\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx\right]$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3}\frac{1}{3}\int e^{3x}\,3\,dx\right]$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right] + C$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

$\int u\,dv$

Ex $\int \ln x \cdot dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \Leftrightarrow v = x$

$= x\ln x - \int x \cdot \frac{1}{x}\,dx$

$= x\ln x - \int dx$

$= x\ln x - x + c$

$= x(\ln x - 1) + c$

Ch $\frac{d}{dx}\left(x\cdot(\ln x - 1)\right) = x\cdot\left(\frac{1}{x} - 0\right) + (\ln x - 1)\cdot 1$

$= 1 + \ln x - 1 = \ln x$