M 192 Lect #7 9-19-11

$$\int \tan^{-1}(x) \cdot dx$$

$u = \tan^{-1} x \quad dv = dx$

$du = \frac{1}{1+x^2} dx \quad v = x$

$$= \tan^{-1}(x) \cdot x - \frac{1}{2}\int 2x \cdot \frac{1}{1+x^2} dx$$

$u = 1+x^2$

$du = 2x\, dx$

$$= \tan^{-1}(x) \cdot x - \frac{1}{2}\int \frac{du}{u}$$

$$= \tan^{-1}(x) \cdot x - \frac{1}{2} \ln|u| + C$$

$$= x \tan^{-1}(x) - \frac{1}{2} \ln|1+x^2| + C$$

$$= x \tan^{-1}(x) - \frac{1}{2} \ln(1+x^2) + C$$

Trigonometric Integrals

Recall

$$d \sin x = \cos x \, dx$$

$$d \cos x = -\sin x \, dx$$

$$d \tan x = \sec^2 x \, dx$$

$$d \cot x = -\csc^2 x \, dx$$

$$d \sec x = \sec x \tan x \, dx$$

$$d \csc x = -\csc x \cot x \, dx$$

$$\int \sin^5 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx \leftarrow 1^{st}$

$$= \int u^5 \, du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{\sin^6 x}{6} + C$$

$$\int \sin^{10} x \, \cos^3 x \, dx$$

$$= \int \sin^{10} x \, \cos^2 x \, \underbrace{\cos x \, dx}_{du}$$

$$= \int u^{10}(1-u^2)\, du$$

$$= \int (u^{10} - u^{12})\, du$$

$$= \frac{u^{11}}{11} - \frac{u^{13}}{13} + C$$

$$= \frac{\sin^{11} x}{11} - \frac{\sin^{13} x}{13} + C$$

Poem

sin, cos, tan odd
not hard

$u = \sin x$

$du = \cos x \, dx$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$= 1 - u^2$

$$\int \sec^4 x \tan^8 x \, dx$$

Poem

secant even easy

$$= \int \underbrace{\sec^2 x}_{u^2+1} \cdot \underbrace{\tan^8 x}_{u^8} \underbrace{\sec^2 x \, dx}_{du}$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$= \int (u^2+1)\, u^8 \, du$$

$\sec^2 x = \tan^2 x + 1$

$= u^2 + 1$

$$= \int (u^{10} + u^8) \, du$$

$$= \frac{u^{11}}{11} + \frac{u^9}{9} + C$$

$$= \frac{\tan^{11}(x)}{11} + \frac{\tan^9(x)}{9} + C$$