M192 Lect #8 9-21-11

tan odd, not hard

$$\int \sec^3(5x)\tan^3(5x)\,dx$$

keep, convert

$$\frac{1}{5}\int \sec^2(5x)\tan^2(5x)\,\underbrace{\sec(5x)\tan(5x)\,5\,dx}_{du}$$

$$\frac{1}{5}\int u^2(u^2-1)\,du$$

$$\frac{1}{5}\int (u^4-u^2)\,du$$

$$\frac{1}{5}\left(\frac{u^5}{5}-\frac{u^3}{3}\right)+C$$

$$\frac{1}{5}\left(\frac{\sec^5(5x)}{5}-\frac{\sec^3(5x)}{3}\right)+C$$

$u = \sec(5x)$ keep

$du = \sec(5x)\tan(5x)\,5\,dx$

$\sec^2 = \tan^2+1$

$\tan^2(5x) = \sec^2(5x)-1$

$= u^2-1$

Our last major 8.2 type is the sin and cos even problem

$$\int \sin^2(x)\cos^2(x)\,dx$$

**half angle identity**

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$= \int \frac{1-\cos(2x)}{2}\cdot\frac{1+\cos(2x)}{2}\,dx$$

$$= \frac{1}{2^2}\int \left(1-\cos^2(2x)\right)dx$$

$$= \frac{1}{4}\int \left(1\cdot\frac{2}{2} - \frac{1+\cos(4x)}{2}\right)dx$$

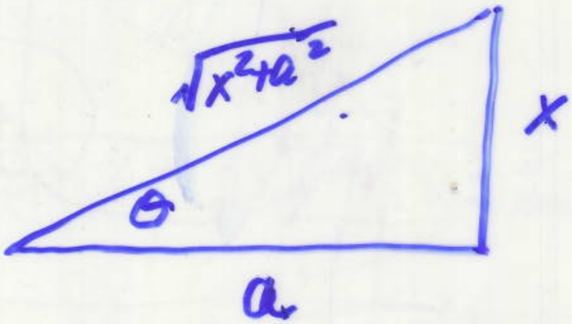
$$= \frac{1}{8}\left[\int 1\,dx - \frac{1}{4}\int \cos(4x)\,4\,dx\right]$$

$u = 4x$
$du = 4\,dx$

$$= \frac{1}{8}\left[x - \frac{1}{4}\sin(4x)\right] + C = \frac{1}{32}\left[4x - \sin(4x)\right] + C$$

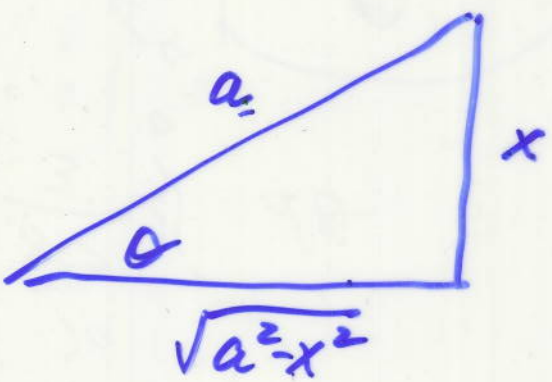
# §8.3. Trig Substitution

$$\frac{x}{a} = \tan\theta$$




$$\boxed{\begin{aligned} x &= a\tan\theta \\ dx &= a\sec^2\theta\, d\theta \\ \sqrt{x^2+a^2} &= a\sec\theta \end{aligned}}$$

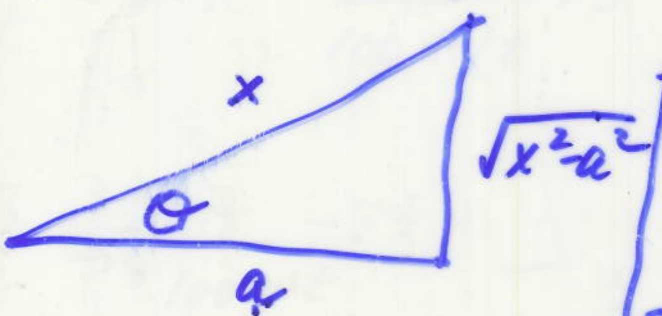
$$\frac{x}{a} = \sin\theta$$




$$\boxed{\begin{aligned} x &= a\sin\theta \\ dx &= a\cos\theta\, d\theta \\ \sqrt{a^2-x^2} &= a\cos\theta \end{aligned}}$$

$$\frac{\sqrt{\phantom{x}}}{a} = \cos\theta$$

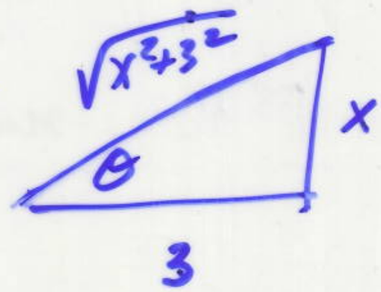
$$\frac{x}{a} = \sec\theta$$




$$\boxed{\begin{aligned} x &= a\sec\theta \\ dx &= a\sec\theta\tan\theta\, d\theta \\ \sqrt{x^2-a^2} &= a\tan\theta \end{aligned}}$$

$$\frac{\sqrt{\phantom{x}}}{a} = \tan\theta$$

Use Trig Subst to integrate




$$\int \sqrt{x^2+9}\, x^3\, dx$$

$\frac{x}{3} = \tan\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\frac{\sqrt{\phantom{x}}}{3}$

$\sqrt{x^2+9} = 3\sec\theta$

$$\int 3\sec\theta\,(3\tan\theta)^3\, 3\sec^2\theta\, d\theta$$

$$\int 3^5 \sec^3\theta \tan^3\theta\, d\theta$$

$$3^5 \int \sec^2\theta \tan^2\theta\, \underbrace{\sec\theta\tan\theta\, d\theta}_{du}$$

$u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

convert $\tan^2\theta = \sec^2\theta - 1$

$= u^2 - 1$

$$3^5 \int u^2(u^2-1)\, du$$

$$3^5 \int (u^4 - u^2)\, du$$

$$3^5 \left[\frac{u^5}{5} - \frac{u^3}{3}\right] + C$$

$$= 3^5\left[\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3}\right] + C$$

$$= 3^5\left[\frac{1}{5}\left(\frac{\sqrt{x^2+9}}{3}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{x^2+9}}{3}\right)^3\right] + C$$

$$= \frac{1}{5}\cdot \cancel{3^5}\frac{(x^2+9)^{5/2}}{\cancel{3^5}} - \frac{1}{3}\cdot 3^5\,\frac{(x^2+9)^{3/2}}{3^3} + C$$

$$= \frac{1}{5}(x^2+9)^{5/2} - 3(x^2+9)^{3/2} + C$$

$$= \left[\frac{1}{5}(x^2+9)^1 - 3\cdot\frac{5}{5}\right](x^2+9)^{3/2} + C$$

$$= (x^2+9-15)\,\frac{(x^2+9)^{3/2}}{5}$$

$$(x^2-6)\,\frac{(x^2+9)^{3/2}}{5} + C$$