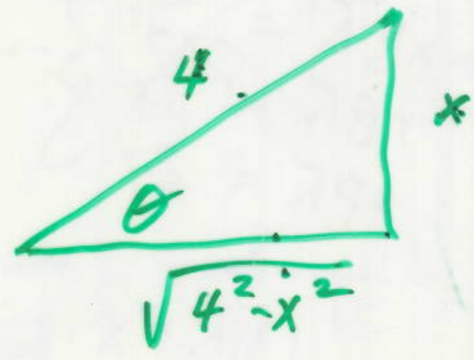
M 192 Lect #9 9-26-11

Again a trig subst problem

$$\int \frac{x^5}{(16-x^2)^{3/2}}\,dx$$



$$\frac{x^5}{\sqrt{4^2-x^2}^{\,3}}\,dx$$

$\frac{x}{a} = \frac{\text{opp}}{\text{hyp}} = \sin\theta$

$$\boxed{\begin{aligned} x &= 4\sin\theta \\ dx &= 4\cos\theta\,d\theta \\ \sqrt{4^2-x^2} &= 4\cos\theta \end{aligned}}$$

$$= \int \frac{4^5 \sin^5\theta}{4^3\cos^3\theta}\,4\cos\theta\,d\theta$$

$$= 4^3 \int \frac{\sin^5\theta}{\cos^2\theta}\,d\theta$$

$\frac{\sqrt{16-x^2}}{4} = \cos\theta$

sin odd, not hard

convert

$$= -4^3 \int \frac{\sin^4\theta}{\cos^2\theta}\underbrace{(-\sin\theta\,d\theta)}_{du}$$

keep

$u = \cos\theta$ ← keep

$du = -\sin\theta\,d\theta$

$\sin^2\theta = 1-\cos^2\theta$

$= 1-u^2$

$$= -4^3 \int \frac{(1-u^2)^2}{u^2}\,du$$

$$= -4^3 \int \frac{1-2u^2+u^4}{u^2}\,du$$

$$= -4^3 \int \left(u^{-2} - 2 + u^2\right)du$$

$$= -4^3 \left[\frac{u^{-1}}{-1} - 2u + \frac{u^3}{3}\right] + C$$

$$= -4^3 \left[-\frac{1}{\cos\theta} - 2\cos\theta + \frac{\cos^3\theta}{3}\right] + C$$

$$= -4^3 \left[-\frac{4}{\sqrt{16-x^2}} - 2\,\frac{\sqrt{16-x^2}}{4} + \frac{1}{3}\frac{\sqrt{16-x^2}^{\,3}}{4^3}\right] + C$$

pretty good answer.

8.4 Partial Fractions Decomposition

Recall how we add fractions in algebra

$$\frac{x+2}{x+2}\cdot\frac{2}{x-5} + \frac{6}{x+2}\cdot\frac{x-5}{x-5} = \frac{2(x+2)+6(x-5)}{(x+2)(x-5)}$$

$$= \frac{2x+4+6x-30}{x^2-3x-10} = \frac{8x-26}{x^2-3x-10}$$

Integrate

$$\int \frac{8x-26}{x^2-3x-10}\,dx$$

$$\frac{8x-26}{(x-5)(x+2)} = \frac{(x+2)}{(x+2)}\cdot\frac{A}{x-5} + \frac{B}{x+2}\cdot\frac{(x-5)}{(x-5)}$$

$$= \frac{A(x+2)+B(x-5)}{(x-5)(x+2)}$$

$$8x-26 = A(x+2) + B(x-5)$$

$x=-2 \Rightarrow -16-26 = A\cdot 0 + B(-7) \Rightarrow -42 = -7B$

$\Rightarrow B = 6$

$x=5 \Rightarrow 40-26 = A(7) + B(0) \Rightarrow 14 = 7A$

$\Rightarrow A = 2$

$$\text{So} \int \frac{8x-26}{x^2-3x-10}\,dx = \int \frac{2}{x-5}\,dx + \int \frac{6}{x+2}\,dx$$

$u = x-5$
$du = dx$

$v = x+2$
$dv = dx$

$$= 2\int \frac{du}{u} + 6\int \frac{dv}{v}$$

$$= 2 \ln|u| + 6 \ln|v| + C$$

$$= 2 \ln|x-5| + 6 \ln|x+2| + C$$

$$= \ln|x-5|^2 + \ln|x+2|^6 + C$$

$$= \ln\left(|x-5|^2 \cdot |x+2|^6\right) + C$$

$$= \ln (x-5)^2(x+2)^6 + C$$

Short Cut for rational fcns with linear factors to first power in denom

$$\frac{8x-26}{(x-5)^1(x+2)^1} = \frac{\cancel{\frac{14}{7}}}{x-5} + \frac{\cancel{\frac{-42}{-7}}}{x+2}$$

$$5 \qquad\qquad -2$$

$$= \frac{2}{x-5} + \frac{6}{x+2}$$

$$\int \frac{5}{(x-1)\,x\,(x+4)}\,dx = \frac{\cancel{\frac{5}{(1)(5)}}}{x-1} + \frac{\cancel{\frac{5}{(-1)(4)}}}{x} + \frac{\cancel{\frac{5}{(-5)(-4)}}}{x+4}$$

$$1 \qquad\qquad 0 \qquad\qquad -4$$

$$= \int\left(\frac{1}{x-1} - \frac{5}{4}\,\frac{1}{x} + \frac{1}{4}\,\frac{1}{x+4}\right)dx$$

$$= \frac{4}{4}\ln|x-1| \quad \frac{5}{4}\ln|x| + \frac{1}{4}\ln|x+4| + C$$

$$= \frac{1}{4} \ln \left| \frac{(x-1)^4 \cdot (x+4)^1}{x^5} \right| + C$$