M192 Lect #10 9-28-11

most general proper fraction

2nd P.F. example

$$\int \frac{2x-1}{x^3(x+4)}\,dx$$

$$\frac{2x-1}{x^3(x+4)} = \frac{(Ax^2+Bx+C)(x+4)}{x^3\,(x+4)} + \frac{Dx\cdot x^3}{x+4\;x^3}$$

$$= \frac{(Ax^2+Bx+C)(x+4) + Dx^3}{x^3(x+4)}$$

$$2x-1 = (Ax^2+Bx+C)(x+4) + Dx^3$$

$x=-4$

$x=0$

$x=1$

$x=2$

$$\int\left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+4}\right)dx$$

$$A\ln|x| + B\frac{x^{-1}}{-1} + C\frac{x^{-2}}{-2} + D\ln|x+4|$$

3rd P. F. example

$$\int \frac{x+5}{(x-3)(x^2+9)}\,dx = \int \left(\frac{A}{x-3} + \frac{Bx+C}{x^2+9} \right) dx$$

$x=3$

$x=0$

$x=1$

4th P. F. example

Tempted Ax^2+Bx+C

$$\int \frac{7}{(x-6)^3\,x} = \frac{A(x-6)^2+B(x-6)+C}{(x-6)^3} + \frac{D}{x}$$

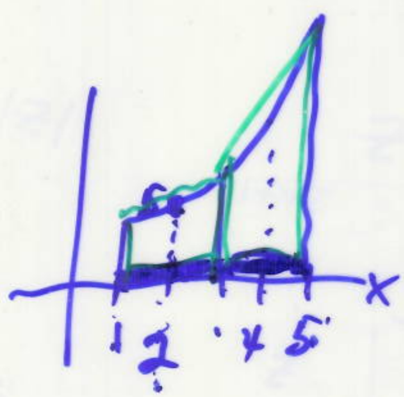
Numerical Integration

We need a table

① If $f(x) = x^2+1$ on $[1,5]$

$n = 2$ intervals

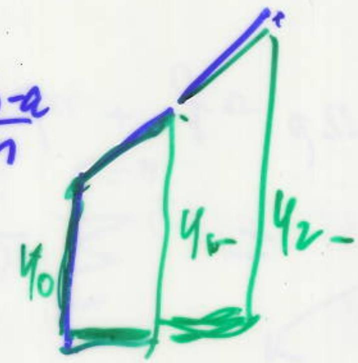
② If not $f(x)$ given you measure to get table data



$$\frac{5-1}{2} = \frac{b-a}{n}$$

$2 = h$

$2 = \Delta x$



MP	EP (x)	y	MP	TR	SR
	1	2		y_0	y_0
2	2	5 ←	y_1		y_1
	3	10		y_1	y_2
4		17 ←	y_2		y_3
	5	26		y_2	y_4

$h=1$

$2=h$

$$M_2 = \Delta x \left(1y_1 + 1y_2\right)$$

$1 \quad 2 \quad 2 \quad 2 \quad 2 \cdots 2 \quad 1$

$$T_2 = \Delta x\left(\frac{y_0+y_1}{2} + \frac{y_1+y_2}{2}\right) = \frac{\Delta x}{2}\left(y_0 + 2y_1 + 1y_2\right)$$

$$T_6 = \frac{\Delta x}{2}\left(1y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + 1y_6\right)$$

new & smaller

$$S_4 = \frac{\Delta x}{3}\left(1y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$S_8 = \frac{\Delta x}{3}\left(1y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_0 + 4y_7 + 1y_8\right)$$

$$S_4 = \frac{2M_2 + 1T_2}{3}$$

Use Simpson's Rule to find an approximation for the integral of $f(x)$ on $[10, 30] = [a, b]$ defined by this table

$$\int_{10}^{30} f(x)\,dx$$

$n = 10$ intervals

x	y
10	3
12	4
14	5
16	4
18	4
20	3
22	2
24	2
26	2
28	3
30	5

$h = [2$

$$h = \Delta x = \frac{30-10}{10} = \frac{20}{10} = 2$$

$$S_{10} = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 \text{ etc} \quad 4y_9 + 1y_{10}\right)$$

$$= \frac{2}{3}\left[3 + 16 + 10 + 16 + 8 + 12 + 4 + 8 + 4 + 12 + 5\right]$$

$$= \frac{2}{3}\left[3 + 5 + 4(16) + 2(13)\right]$$

$$= \frac{2}{3}\left[8 + 64 + 26\right]$$

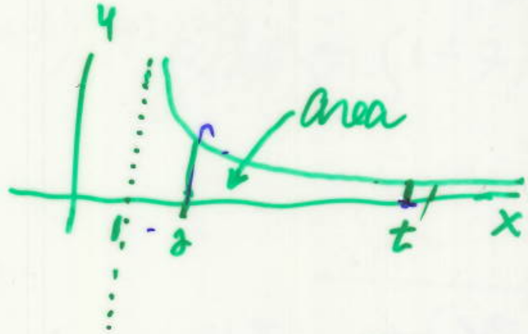
$$= \frac{2}{3}(98) = 65\tfrac{1}{3}$$

Improper Integrals

(1) Infinite limits

(2) Infinite integrand

(1) $$\int_2^{\infty} \frac{1}{x-1}\,dx$$

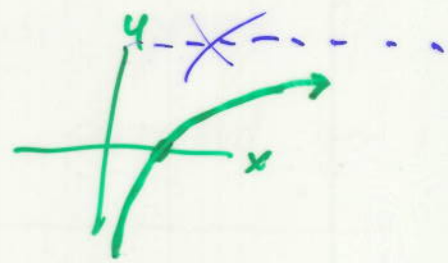


$$= \lim_{t\to\infty} \int_2^{t} \frac{1}{x-1}\,dx$$

$u = x-1$
$du = dx$

$$\int \frac{du}{u} = \ln|u|$$

$$= \lim_{t\to\infty} \ln|x-1| \Big]_2^{t}$$



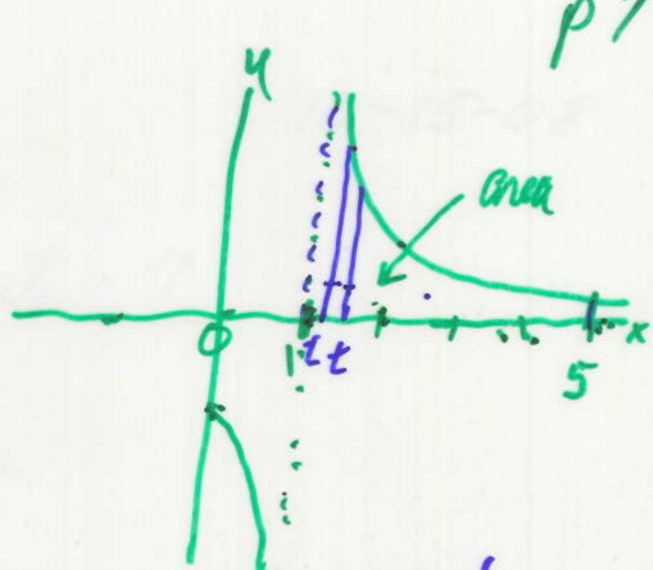
$$= \lim_{t\to\infty} \left[\underbrace{\ln(t-1)}_{\infty} - \underbrace{\ln(2-1)}_{0}\right]$$

$= \infty$

We write $\int_2^{\infty} \frac{1}{x-1}\,dx$ diverges (to infinity)

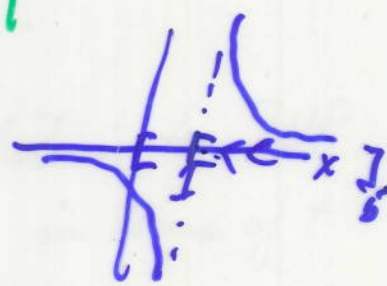
We say the improper integral diverges

② $\int_1^5 \frac{1}{x-1}\,dx$



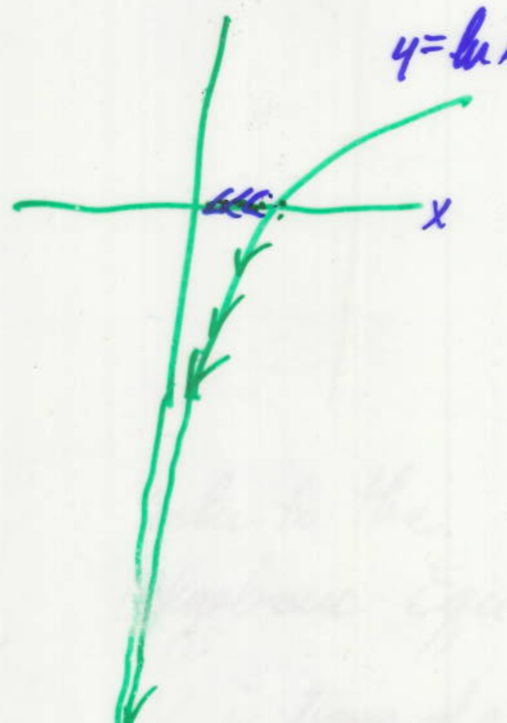
$= \lim_{t\to1^+} \int_t^5 \frac{1}{x-1}\,dx$

bad spot for fcn $\frac{1}{x-1}$



$= \lim_{t\to1^+} \ln(x-1)\Big]_t^5$

$= \lim_{t\to1^+} \left[\overset{1.4}{\ln(5-1)} - \ln(t-1)\right]$



$\doteq 1.4 - \lim_{t\to1^+} (t-1)$

$= 1.4 - (\text{large neg})$

$= \infty$

We conclude that the improper integral diverges p8

$$\int_1^6 \frac{2}{(x-4)^3}\,dx = \int_1^4 \frac{2}{(x-4)^3}\,dx + \int_4^6 \frac{2}{(x-4)^3}\,dx$$

bad @ $x=4$

work one side at a time

$$= \lim_{t \to 4^-} \int_1^t \frac{2}{(x-4)^3}\,dx$$

$$|E_s| \le EB_s \stackrel{def}{=} \frac{K(b-a)^5}{180\, n^4} \quad \text{where}$$

$n=6$

say $n=6$ $\qquad K = \max_{a \le x \le b} |f''''(x)|$

Example

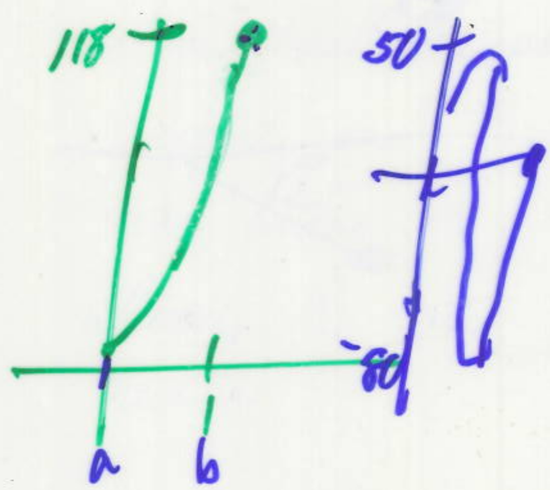
$f(x) = e^{2x}$ $\quad$ on $[0,1]$

$f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

$f''''(x) = 16e^{2x}$



$K \doteq 120 \approx 118$

$$E_s \le EB_s = \frac{120(1-0)^5}{180\; 6^4} = 5.1 \times 10^{-4}$$

$= .00052$