M 192 Lect #11 10-3-11

Start out with an improper fraction (rat fcn) as your integrand.

$$\int \frac{x^3+3x^2-12x+1}{x^2+x-12}\,dx$$

$$\frac{x^3}{x^2} = x$$

$$\frac{2x^2}{x^2} = 2$$

First do long division to get a whole part and a proper rat fcn part

$$\begin{array}{r} x+2 \\ x^2+x-12 \;\overline{)\; x^3+3x^2-12x+1} \\ \text{Subtr} \quad x^3+x^2-12x \\ \hline 2x^2+0x+1 \\ \text{Subtr} \quad 2x^2+2x-24 \\ \hline -2x+25 \end{array}$$

$$\frac{x^3+3x^2-12x+1}{x^2+x-12} = x+2+\frac{-2x+25}{x^2+x-12}$$

$$\text{Orig int} = \int (x+2)\,dx + \int \frac{-2x+25}{x^2+x-12}\,dx$$

$$\frac{-2x+25}{x^2+x-12} = \frac{-2x+25}{(x-3)(x+4)} = \frac{+19/7}{x-3} + \frac{33/-7}{x+4}$$

$3 \qquad -4$

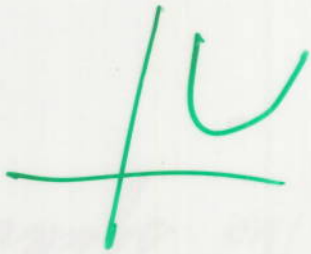
$$\text{Orig int} = \frac{x^2}{2} + 2x + \int \frac{19/7}{x-3}\,dx - \frac{33}{7}\int \frac{1}{x+4}\,dx$$

$$= \frac{x^2}{2} + 2x + \frac{19}{7}\ln|x-3| - \frac{33}{7}\ln|x+4| + C$$

$$= \frac{x^2}{2} + 2x + \frac{1}{7}\ln\left(\frac{|x-3|^{19}}{|x+4|^{33}}\right) + C$$

Ex Irreducible quadratic in denom.

$$\int \frac{6x-8}{x^2-6x+25}\,dx$$



$$\int \frac{6x-8}{x^2-6x+9-9+25}\,dx = \int \frac{6x-8}{(x-3)^2+4^2}\,dx$$

$$= 6\int \frac{(x-3+3)-8}{(x-3)^2+4^2}\,dx = \int \frac{6(x-3)+10}{(x-3)^2+4^2}\,dx$$

$$= \frac{6}{2}\int \frac{2(x-3)}{(x-3)^2+4^2}\,dx + 10\int \frac{1}{(x-3)^2+4^2}\,dx$$

let $u = (x-3)^2+4^2$

$du = 2(x-3)\,dx$

$u = x-3$

$du = dx$ $\quad a = 4$

$$= 3\int \frac{du}{u} + 10\int \frac{1}{u^2+a^2}\,du$$

$$= 3\ln|u| + 10 \cdot \frac{1}{a}\tan^{-1}\frac{u}{a}$$

$$= 3\ln\left[(x-3)^2+4^2\right] + \frac{10}{4}\tan^{-1}\left(\frac{x-3}{4}\right) + C$$

$\ln Q = kt + C$ Apply ①

(Q: 10, t: 0)

$\ln 10 = k \cdot 0 + C \Rightarrow C = \ln 10$

Soln becomes

$\ln Q = kt + \ln 10$

$\ln Q - \ln 10 = kt$

$\ln \frac{Q}{10} = kt$ Apply ②

(Q: 80, t: 3)

$\ln \frac{80}{10} = k \cdot 3$

$k = \frac{\ln 8}{3}$

Soln becomes

$\ln \frac{Q}{10} = \frac{\ln 8}{3} \cdot t$

$e^{\ln \frac{Q}{10}} = e^{\frac{\ln 8}{3} \cdot t}$

$\frac{Q}{10} = e^{\frac{\ln 8}{3} \cdot t}$

$Q = 10\, e^{\frac{\ln 8}{3} \cdot t}$

Solve for t

$t = \frac{\ln \frac{Q}{10}}{\frac{\ln 8}{3}}$

We can ask 3 questions

(1) How much

(2) When will

(3) How fast

(4) Relatively How fast