M 192 Lect #13 10-10-11

The Calculus of Parametric Eqns

$$\begin{cases} x = x(t) = 1+3t \\ y = y(t) = 4+t^2 \end{cases}$$

$$\frac{dx}{dt} = \dot{x} = 3 \qquad dx = \dot{x}\,dt = 3\,dt$$

$$\frac{dy}{dt} = \dot{y} = 2t \qquad dy = \dot{y}\,dt = 2t\,dt$$

$$y' = \frac{dy}{dx} = \frac{\dot{y}\,dt}{\dot{x}\,dt} = \frac{2t\,dt}{3\,dt} = \text{slope of tan line}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right) = \frac{\frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^2}}{\frac{\dot{x}}{1}} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^3}$$

$$\frac{dy}{dx} = \frac{2t}{3} \qquad 6t \qquad 6$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{3}t\right)$$

$$= \frac{\frac{d}{dt}\left(\frac{2}{3}t\right)}{\frac{dx}{dt}} = \frac{2/3}{3} = \frac{2}{9}$$