M 192 Lect #14 10-12-11

Let's finish the last problem by finding the eqn of the tan line @ $t=2$

Ⓐ in Rect. eqns.

a line

$y - y_1 = m(x - x_1)$

$t_1 = 2 \Rightarrow$

$x = 1 + 3t_1 = 7 = x_1$

$y = 4 + t^2 = 8 = y_1$

Slope $y' = \frac{dy}{dx} = \frac{2t}{3}$

$y' = m = \frac{4}{3} = \frac{\text{rise}}{\text{run}}$

Eq of tan line is

$$\boxed{y - 8 = \frac{4}{3}(x - 7)}$$

$y - y_1 = m(x - x_1)$

Ⓑ in Parametric eqns.

a line

$$\begin{cases} x = x_1 + \text{run} \cdot t \\ y = y_1 + \text{rise} \cdot t \end{cases}$$

So we get

$$\begin{cases} x = 7 + 3t \\ y = 8 + 4t \end{cases}$$

Often we'll write the eqs of tan line with another parameter

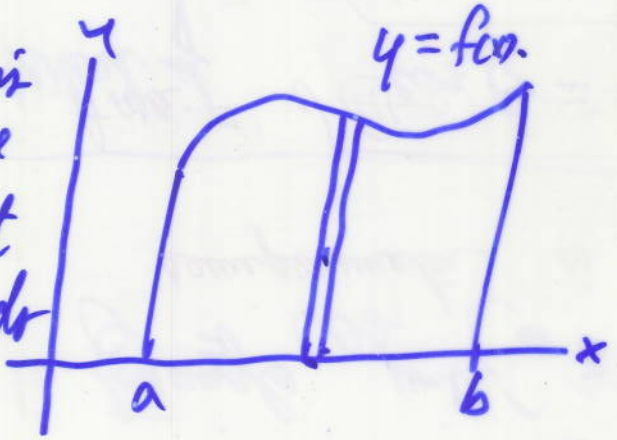
$x = 7 + 3u$

$y = 8 + 4u$

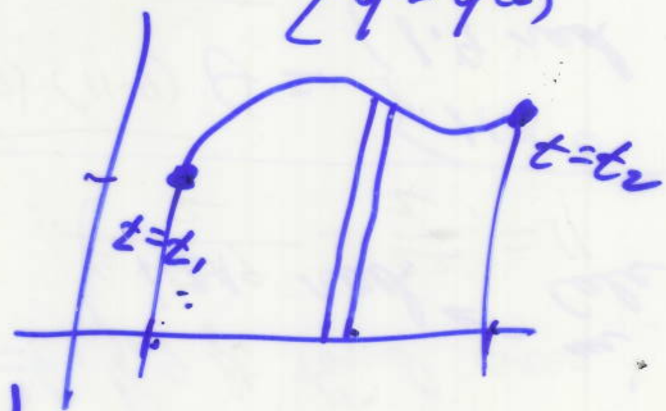
$dx = \dot{x}(t)\,dt$

Areas in Param Eqns

This one rect coords



$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

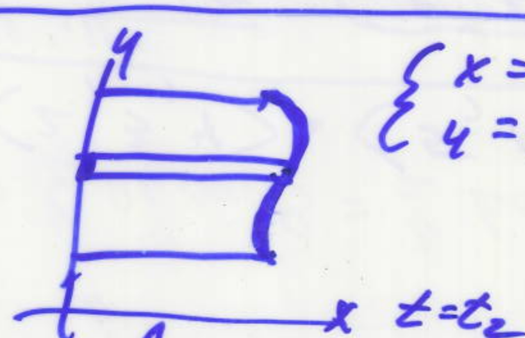


$$A = \int_0^A dA = \int_a^b y\,dx$$

$$= \int_a^b f(x)\,dx$$

$$A = \int_0^A dA = \int_{t=t_1}^{t_2} y\,dx$$

$$= \int_{t=t_1}^{t_2} y(t)\,\dot{x}(t)\,dt$$



$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

$$A = \int_0^A dA = \int_{t=t_1} x\,dy$$

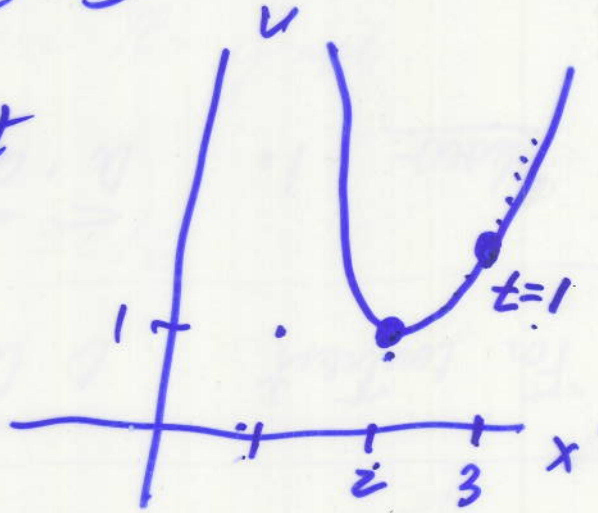
$$= \int_{t_1}^{t_2} x(t)\,\dot{y}(t)\,dt$$

p2

p26

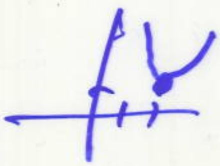
Ex Find the area under this parabola between $t=1$ and $t=3$

$$\begin{cases} x = 2 + t & dx = dt \\ y = 1 + t^2 \end{cases}$$

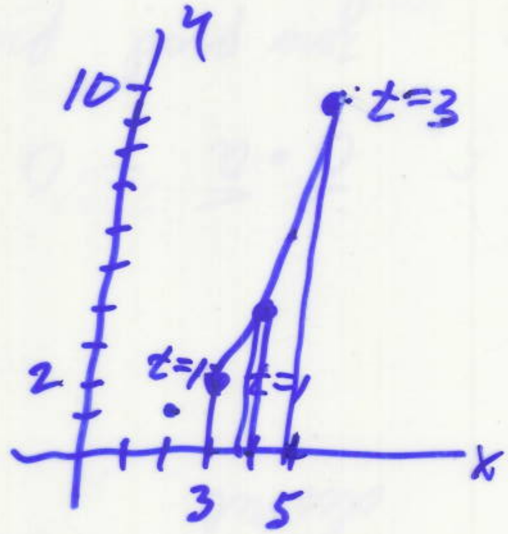


$$\begin{cases} x-2 = t \\ y-1 = t^2 \end{cases}$$

$y-1 = (x-2)^2$



t	x	y
1	3	2
3	5	10

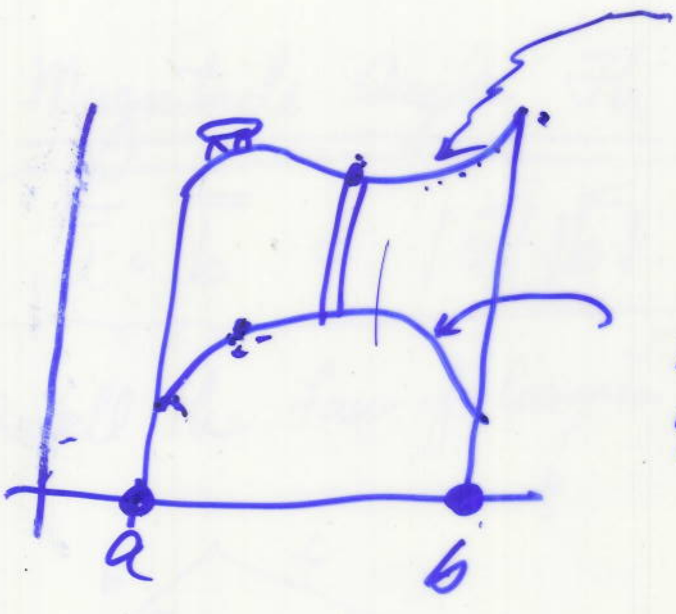


$$A = \int_0^A dA = \int_{t=1}^{3} y\, dx$$

$dx = (0+1)\,dt$

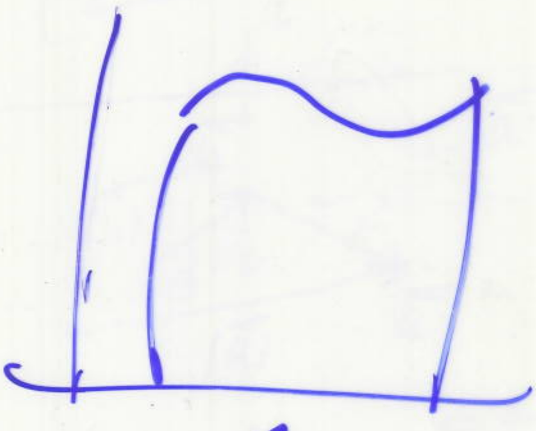
$$= \int_{t=1}^{3} (1+t^2)\,1\,dt = t + \frac{t^3}{3}\Big]_1^3 = 3-1 + \frac{27-1}{3}$$

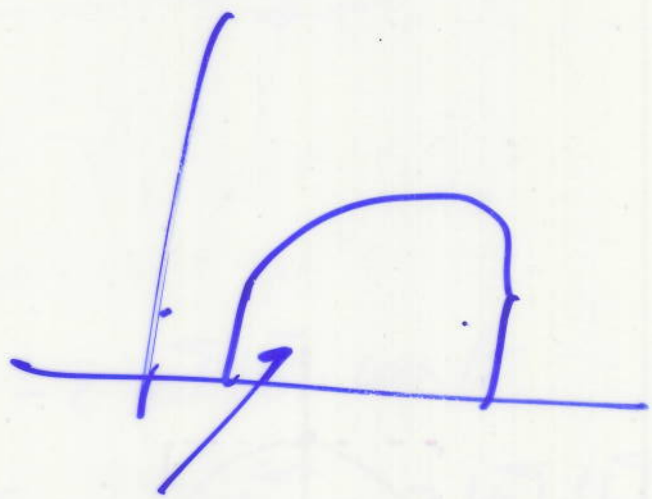
$$= 2 + 8\tfrac{2}{3} = 10\tfrac{2}{3}$$



$x = f(t)$ $\quad 1 \le t \le 3$
$y = g(t)$

$x = h(t)$ $\quad 2 \le t \le 5$
$y = i(t)$





Area 1 − Area 2 = Area

Growth & Decay Problem

Bacteria grows at a rate proportional to the amount present at any time. If there are 3 grams of bacteria initially and 20 grams 5 days later,

a) give the Boundary Value Problem for this Bacteria growth — BVP "DE + BC"

b) Solve it to get a soln.

c) Solve soln for t and B

d) Answer How much
When will
How fast
Relatively how fast questions

Let B = amt of bacteria (grams)
t = time (days)

a) $\boxed{\frac{dB}{dt} = k \cdot B}$, $B(0)=3$, $B(5)=20$

This is BVP or (IVP)

$$\frac{dB}{dt} = kB$$

$$\frac{dB}{B} = k\,dt$$

Since vars are sep'd

$$\int \frac{dB}{B} = \int k\,dt$$

$$\ln|B| = kt + C$$

Apply $B(0)=3$

$$\ln 3 = k0 + C$$

$$C = \ln 3$$

$$\ln B = kt + \ln 3$$

$$\ln B - \ln 3 = kt$$

$$\ln \frac{B}{3} = kt$$

Apply $B(5)=20$

$$\ln \frac{20}{3} = k \cdot 5$$

$$k = \frac{\ln\left(\frac{20}{3}\right)}{5} = \frac{1}{5}\ln\frac{20}{3}$$

$$k = .3794$$

The soln to the IVP is

$$\ln \frac{B}{3} = .3794t$$

e e

c) $\boxed{t = \dfrac{\ln\left(\frac{B}{3}\right)}{.38}} = 5\dfrac{\ln\frac{B}{3}}{\ln\frac{20}{3}}$

$\frac{B}{3} = e^{.38t}$

$\boxed{B = 3e^{.3794\,t}}$

$\boxed{\dfrac{dB}{dt} = .3794\,B}$

t	B	$\frac{dB}{dt}$	$\frac{dB/dt}{B}$
___	1000		
10	___		
		___	

When will (there be 1000 g)

How much bact in 10 days

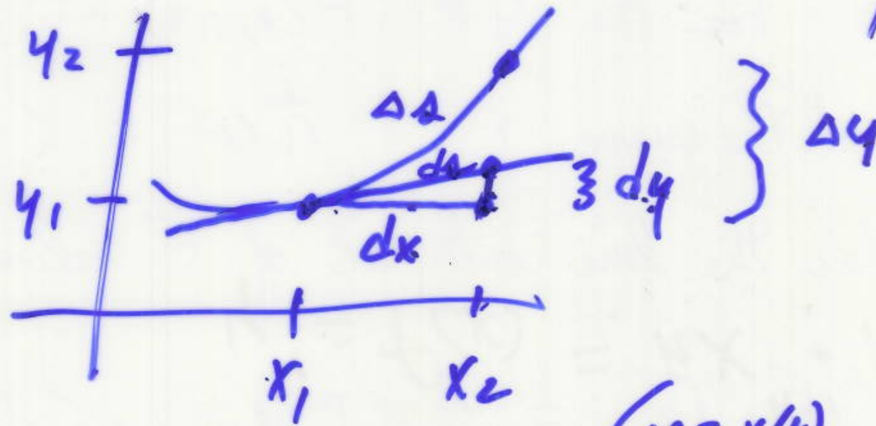
How fast is B changing

Rel how fast

$K = .3794$

___ $\square$ ← 40

Arc length



$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

sum

h

dx

Δx

$$(ds)^2 = (dx)^2 + (dy)^2$$

$$ds = \sqrt{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right](dt)^2}$$

$$s = \sum \Delta s$$

$$s = \int_0^s ds = \int \sqrt{(dx)^2 + (dy)^2}$$

$$\boxed{s = \int_{t=t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt}$$