M 192 Lect #15 10-17-11

We want to find all places on a graph of a parametrically defined curve which have hor & vert tangents

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \qquad \dot{x} = \dot{x}(t) \qquad dx = \dot{x}(t)\,dt$$

$$\dot{y} = \dot{y}(t) \qquad dy = \dot{y}(t)\,dt$$

Hor tan $\frac{dy}{dx}$ is zero or just $dy = 0$

Vert tan $\frac{dy}{dx}$ is undf* or just $dx = 0$

because of being vert

Ex Find Hor & Vert tangents

$$\begin{cases} x = (t-1)^2 + 3 \\ y = (t-4)^2 + 2 \end{cases}$$

$dx = \dot{x}\,dt$
$dy = \dot{y}\,dt$

For Hor $dy \overset{set}{=} 0$

$dy = 2(t-4)\cdot 1\,dt \overset{set}{=} 0$

$t = 4$ prob hor tan

For Vert $dx = 0$

$dx = 2(t-1)\cdot dt \overset{set}{=} 0$

$t = 1$ prob vert tan

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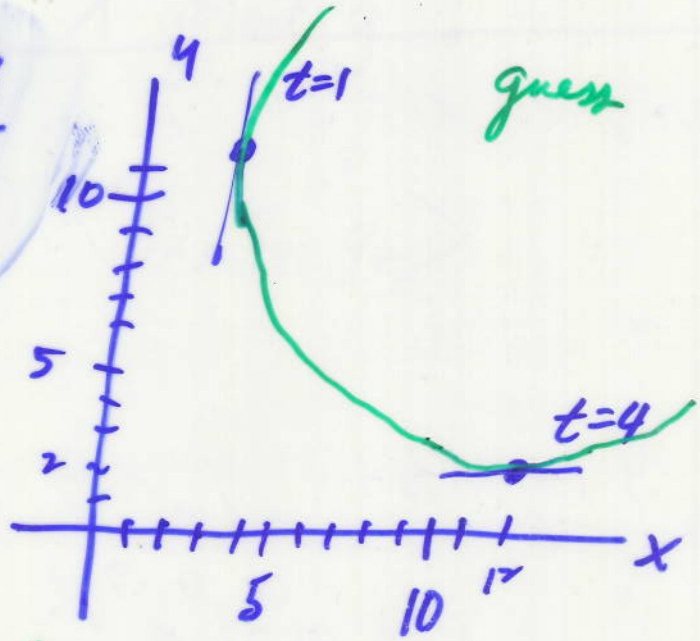
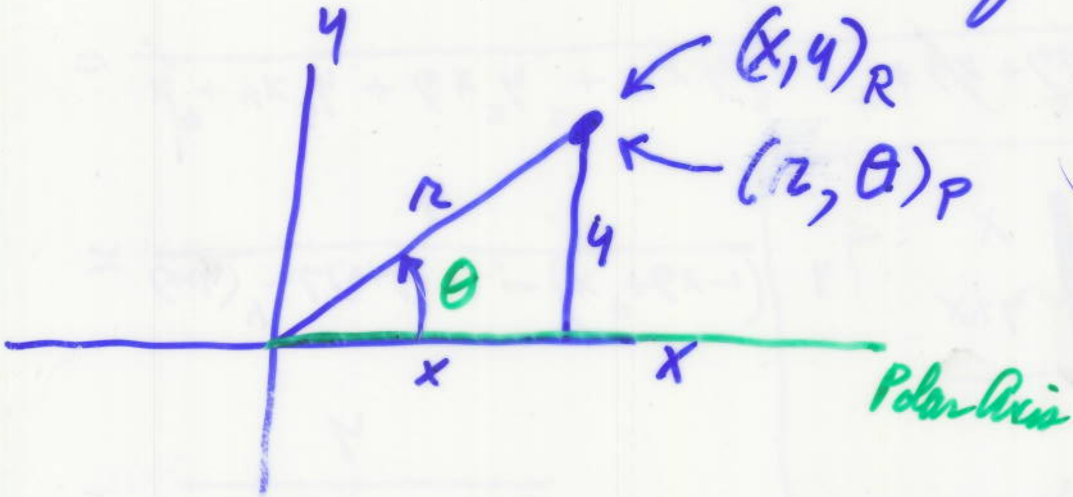
| | $t$ | $x$ | $y$ | $\dot{x}$ | $\dot{y}$ |
|---|---|---|---|---|---|
| V tan | 1 | 3 | 11 | 0 | |
| H tan | 4 | 12 | 2 | | 0 |




Hor tan @ (12, 2) @ time $t=4$

Vert tan @ (3, 11) @ time $t=1$

# Polar Coordinates and Polar Equations




$$\frac{x}{r} = \cos\theta$$

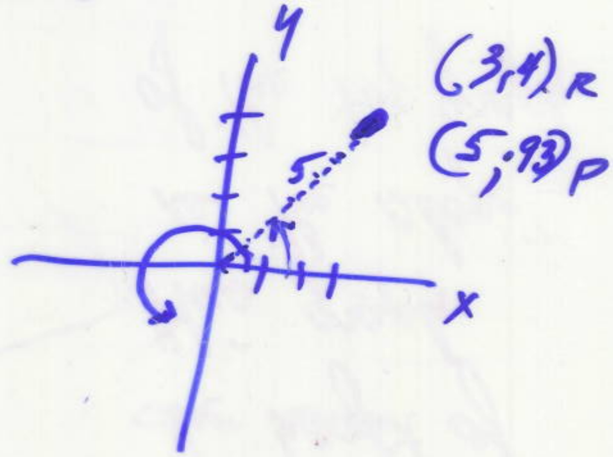
$$x = r\cos\theta$$

Conversion Formulas

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

Convert $(3,4)_R$ to polar

I need $(r, \theta)_P$




$r^2 = x^2 + y^2$

$r^2 = 3^2 + 4^2 = 25$

$r = \pm\sqrt{25} = \pm 5$

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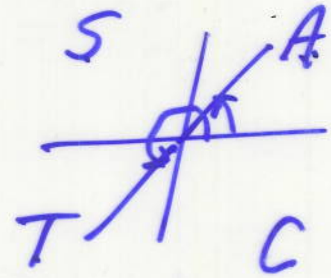


$\tan\theta = \frac{y}{x} = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) + n\pi$ , n an integer

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$(5, .93)_P$

$$\begin{array}{r} 3.14 \\ .93 \\ \hline 4.07 \end{array}$$

$(-5, .93 + \pi)_P = (-5, 4.07)$

plus another infinite # of answers

Convert this eqn in Rect coords to Polar Coords

$$x^2 + y^2 - 3x - \frac{y}{x} = 10$$

$$\boxed{r^2 - 3r\cos\theta - \tan\theta = 10}$$

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$$\left[ r + 5\sin\theta = \frac{3}{r} \right] \times r$$

$$r^2 + 5r\sin\theta = 3$$

$$\boxed{x^2 + y^2 + 5y = 3}$$ $r \neq 0$

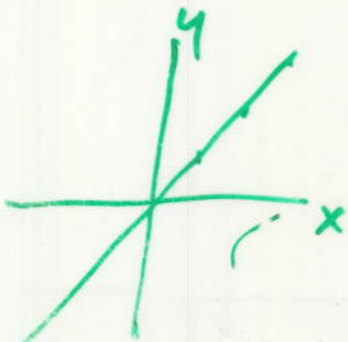
# The Calculus of Polar functions

Just like Rect Coords

$y = f(x)$ we'll write $r = f(\theta)$

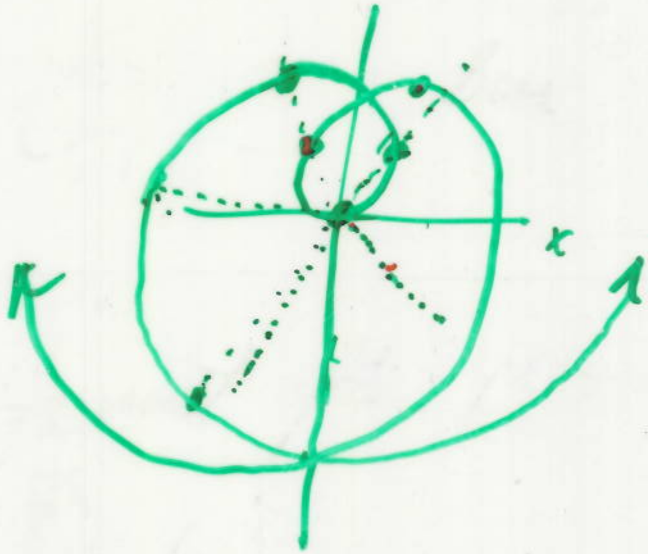
$y = x$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |




$r = \theta$

| $\theta$ | r |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| -1 | -1 |




$\lim_{\theta \to \alpha} f(\theta)$ same as calc I

$\frac{dr}{d\theta} = f'(\theta) = \frac{d}{d\theta}\theta = 1 = \frac{1}{1}$ not the slope of tan line

Change to rect

$\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right)$ a litte

Slope of tan line of $r = f(\theta)$

$$\frac{dy}{dx} \qquad y = r \cdot \sin\theta$$

$$x = r\cos\theta$$

$$\frac{dy}{dx} = \frac{r\cos\theta \cdot \frac{d\theta}{d\theta} + \sin\theta \frac{dr}{d\theta}}{r(-\sin\theta) \cdot \frac{d\theta}{d\theta} + \cos\theta \frac{dr}{d\theta}}$$

$$\boxed{\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}}$$