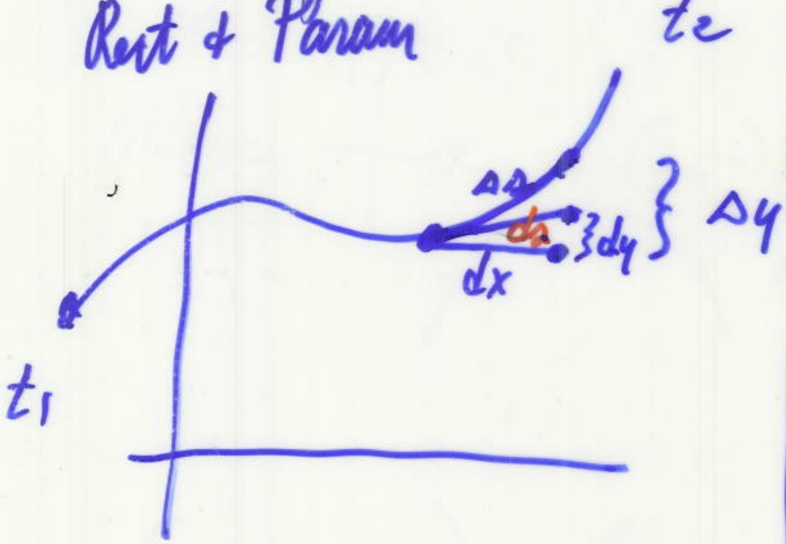
Arc Length
Rect + Param



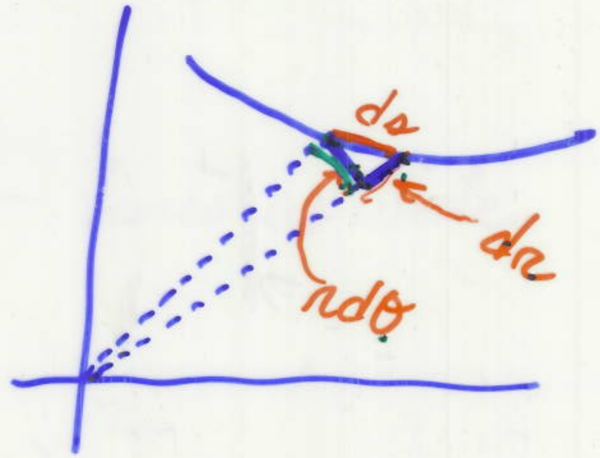
$$s = \int ds = \int \sqrt{(dx)^2+(dy)^2}$$

$$= \int \sqrt{\left(\frac{(dx)^2}{(dt)^2}+\frac{(dy)^2}{(dt)^2}\right)(dt)^2}$$

$$= \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\; dt$$

$$= \int_{t=t_1}^{t_2} \sqrt{\dot{x}^2+\dot{y}^2}\; dt$$

Arc Length
Polar



$$s = \int ds$$

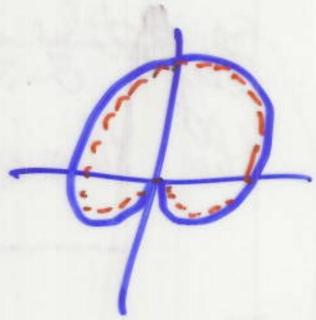
$$= \int \sqrt{\left(\frac{(r\,d\theta)^2}{d\theta^2}+\frac{(dr)^2}{d\theta^2}\right)d\theta^2}$$

$$= \int_{\theta=\alpha}^{\beta} \sqrt{r^2+(r')^2}\; d\theta$$

For the cardioid $r = f(\theta) = 1 + \sin\theta$

$$r' = f'(\theta) = \cos\theta$$

Find length of the perimeter



$$s = \int ds = \int_{\alpha}^{\beta} \sqrt{r^2 + (r')^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\, d\theta$$

$$= 8$$

p 3

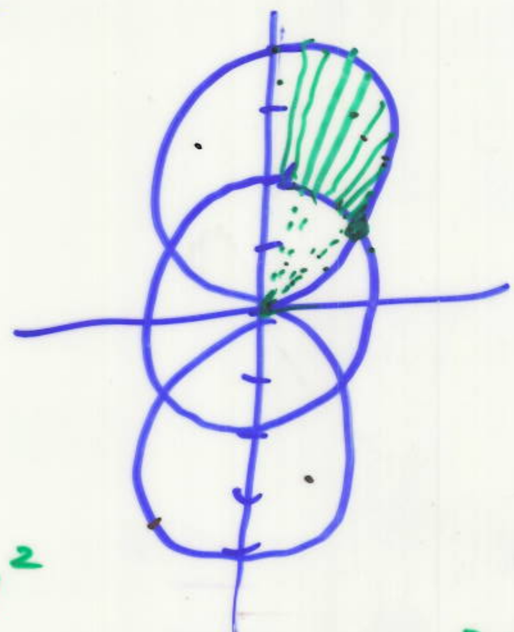
Find the area inside of the lemniscate 8

$r^2 = 16 \sin\theta$ and outside the circle $r = 2$

$r^2 = 4$

By calculator

we need $r = \pm 4\sqrt{\sin\theta}$



$$A = \int dA$$

$$= \int_{\theta=\_}^{\pi/2} \frac{1}{2}\left(r_o^2 - r_i^2\right) d\theta$$

$$= 4\int_{\theta=.2527}^{\pi/2} \frac{1}{2}\left(\overbrace{16\sin\theta}^{r_o^2} - \overbrace{4}^{2^2}\right) d\theta$$

$r_o^2 \overset{set}{=} r_i^2$

$16 \sin\theta \overset{set}{=} 4$

$\sin\theta = \frac{1}{4} = .25$

$\theta = \sin^{-1}(.25)$

$\theta = .2527$

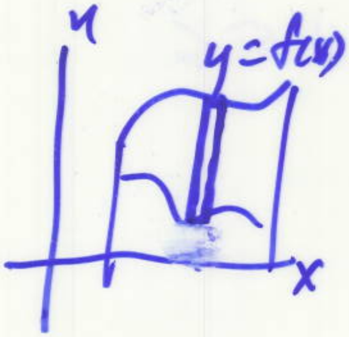
Math 9

fnInt(1/2 * (16 * sin(X) - 4), X, .2527, π/2)

$4(5.1) = 20.4389$

Areas

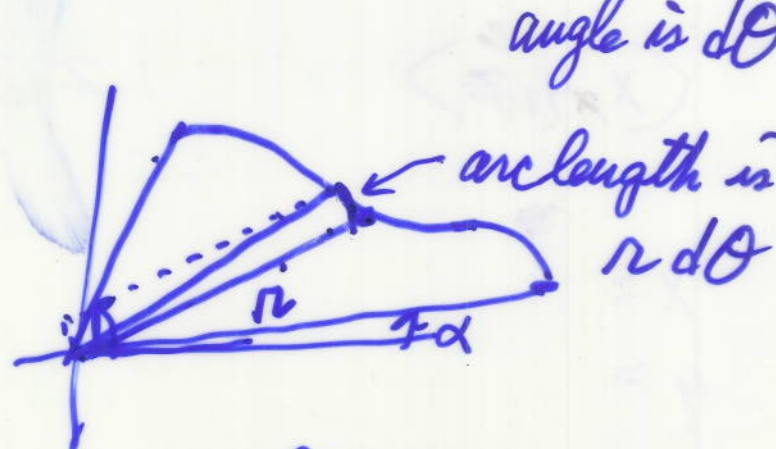
Rect + Param



$$A = \int dA$$

$$= \int (y_t - y_b)\, dx$$

Polar

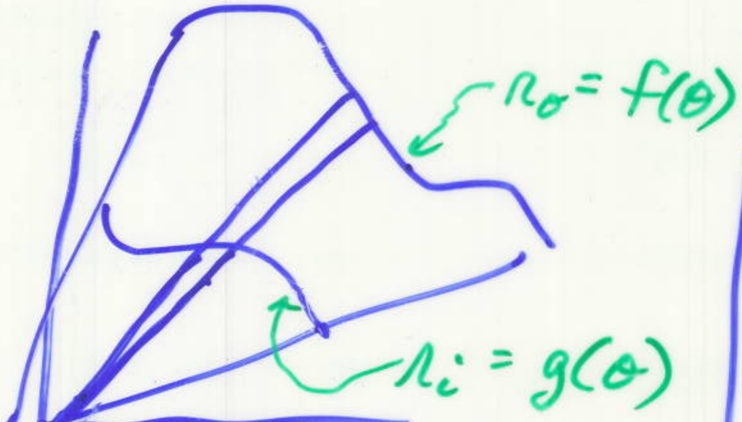


$$A = \int dA$$

$$= \int \frac{1}{2} \text{ base} \times \text{ht}$$

$$= \int \frac{1}{2} r\, r\, d\theta$$

$$= \int_{\alpha}^{\beta} \frac{1}{2} r^2\, d\theta$$



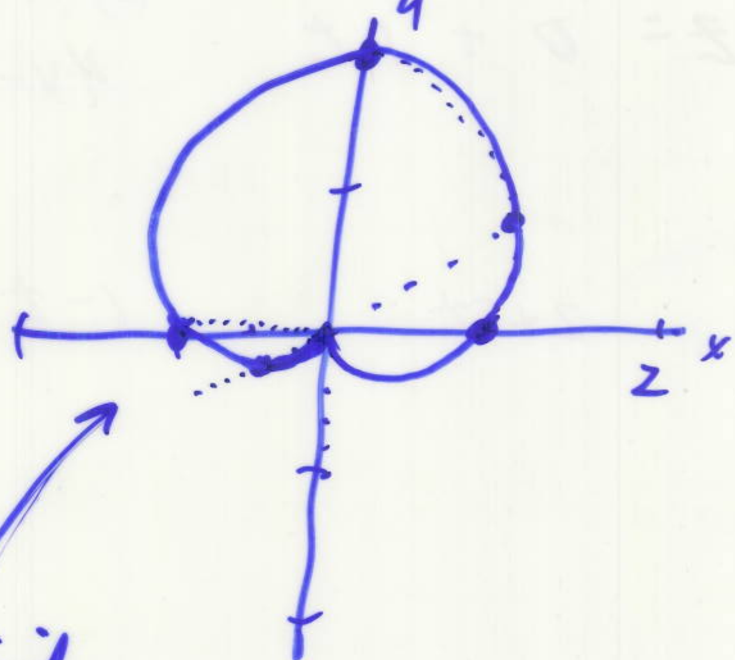
$$A = \int dA = \int_{\alpha}^{\beta} \frac{1}{2} r_o^2\, d\theta - \int_{\alpha}^{\beta} \frac{1}{2} r_i^2\, d\theta$$

$$= \frac{1}{2} \int_{\alpha}^{\beta} (r_o^2 - r_i^2)\, d\theta \;\overset{\text{not}}{=}\; (r_o - r_i)^2$$

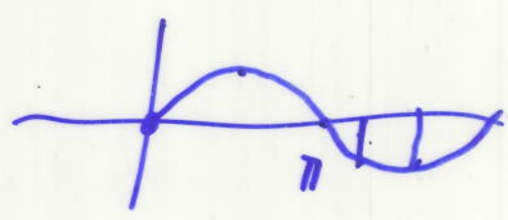
M192 Lect #16 10-19-1?

Let's graph another polar eqn $r = f(\theta) = 1 + \sin\theta$

θ	r
0	1
$\frac{\pi}{6}$	1.5
$\frac{\pi}{2}$	2
π	1
$\frac{7\pi}{6}$	$.5$
$\frac{3\pi}{2}$	0



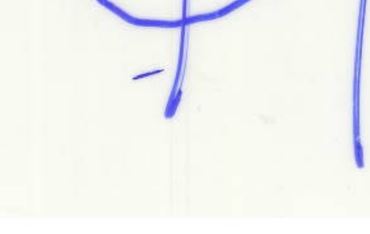
cardioid



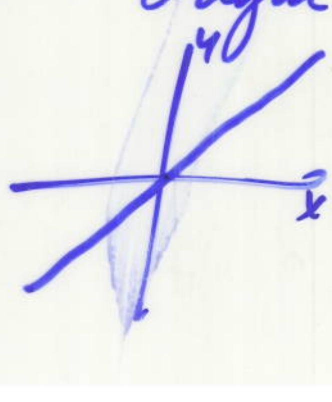
Circle

$r = 3$

$r' = 0$



Line through Origin



$\theta = \frac{\pi}{4}$