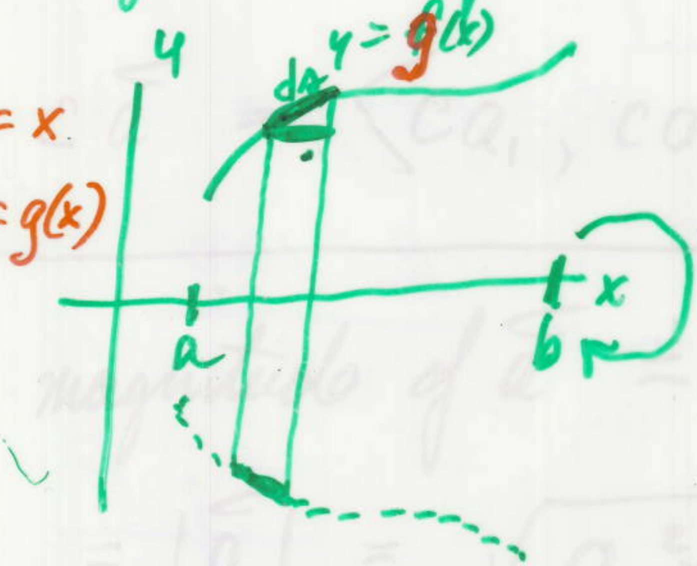
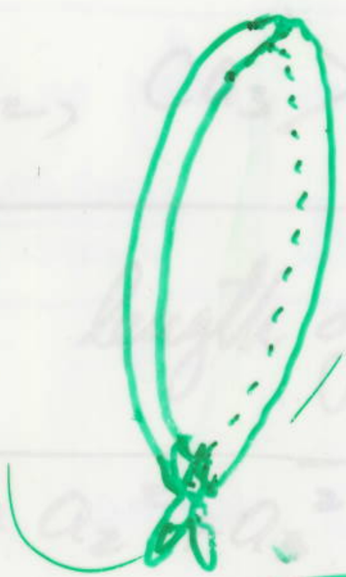
M 192 Lect #17 10-24-11

Surface Area of a Solid of Revolution

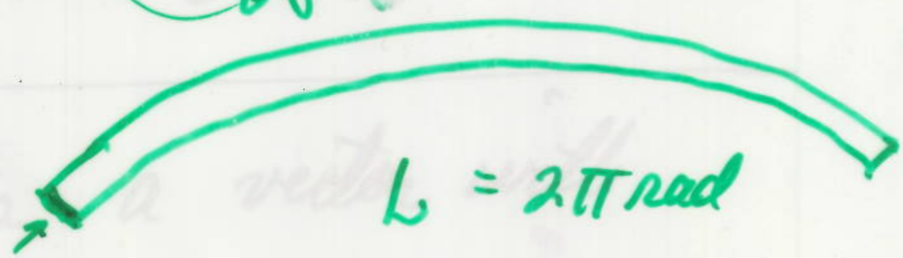
$\begin{cases} x = x \\ y = g(x) \end{cases}$





$C = 2\pi\ rad$

$\begin{cases} x = f(t) = x(t) \\ y = g(t) = y(t) \end{cases}$



$r = f(\theta) = r(\theta)$

ds not dx

$L = 2\pi\ rad$

$L = 2\pi y$

$dS = \text{len} \times \underline{\text{wid}} = 2\pi y\, ds$

Rect	Param	Polar
$dS = 2\pi y\, ds$	$dS = 2\pi y(t)\, ds$	$dS = 2\pi r \sin\theta\, ds$
$= 2\pi y(x) \sqrt{1^2 + (y')^2}\, dx$	$= 2\pi y(t) \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$	$= 2\pi r(\theta) \sin\theta \sqrt{r^2 + (r')^2}\, d\theta$

Chap 12 Infinite Sequences + Infinite Series

12.1 — 12.2 – 12.11

A Sequence is a fcn whose domain is the set of natural numbers (or whole numbers)

$$f(n) = a_n$$

$$\{f(n)\} = \{a_n\}_{n=1}^{\infty} = \{a_n\}$$

Ex $\left\{1 + \frac{2}{n}\right\}_{n=1}^{\infty}$

$$= 1+\frac{2}{1},\ 1+\frac{2}{2},\ 1+\frac{2}{3},\ 1+\frac{2}{4},\ \ldots$$

$$= 3,\ 2,\ 1.\bar{6},\ 1.5,\ \ldots$$

Our interest is in seeing what

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left\{1+\frac{2}{n}\right\} = 1$$

Ex (a) Comm. dots + use calc 1

$$\left\{ \frac{2n-3n^2}{4n+5n^2} \right\}$$ Change all n's to x's

$$f(x) = \frac{2x-3x^2}{4x+5x^2} =$$

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\frac{2x}{x^2} - \frac{3x^2}{x^2}}{\frac{4x}{x^2} + \frac{5x^2}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\overset{0}{\cancel{\frac{2}{x}}} - 3}{\underset{0}{\cancel{\frac{4}{x}}} + 5} = -\frac{3}{5}$$ Limit is $-\frac{3}{5}$

$\boxed{\text{Seq converges to } -3/5}$.

Ex (b) Comm dots + use l'Hop

$$\{a_n\} = \left\{ \frac{3n-4}{8n+2} \right\} = \{f(n)\} \qquad f(x) = \frac{3x-4}{8x+2}$$

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x-4}{8x+2} =$$

$$= \lim_{x\to\infty} \frac{3}{8} = \frac{3}{8}$$

ⓒ Geometric Sequence

$$\{a_n\} = \{a r^n\}_{n=0}^{\infty}$$

$a,\ ar,\ ar^2,\ ar^3, \cdots,\ ar^n, \cdots$

The G Seq converges to 0 $\quad -1 < r < +1$

conv to a $\quad r = 1$

div otherwise

ⓓ $\{a_n\} = \left\{\dfrac{1}{n^p}\right\}_{n=1}^{\infty}$ P-sequence

$\dfrac{1}{1^p}, \dfrac{1}{2^p}, \dfrac{1}{3^p}, \dfrac{1}{4^p}, \cdots, \dfrac{1}{n^p}, \cdots$

p seq conv to 0 for $p > 0$

conv to 1 for $p = 0$

div for $p < 0$

Skip to (h) Recursively defined seq

$a_1 = 5$

$a_2 = e^{\frac{a_1}{4}}$

$a_3 = e^{\frac{a_2}{4}} = e^{\frac{e^{5/4}}{4}}$

$\vdots$

$a_n = e^{\frac{a_{n-1}}{4}}$

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5 sto x ↵

$e^{\wedge}(\frac{x}{4})$ sto x ↵