M 192 Lect #19 10/31/11

We now will begin our thoughts with some sequence called the seq of terms

$$\{a_n\}_{n=1 \leftarrow \text{or } n=0}^{\infty} = \left\{\frac{1}{2^n}\right\}_{n=1}^{\infty} = \left\{1\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty}$$

From this seq of terms we will create a new seq (the sequence of partial sums)

Terms: a_1, a_2, a_3, a_4, ...

Terms: $\frac{1}{2}$, $\frac{1}{2^2}$, $\frac{1}{2^3}$, $\frac{1}{2^4}$, ...

Sums: s_1, s_2, s_3, s_4, ...

Sum $\frac{1}{2^1}$, $\frac{1}{2^1}+\frac{1}{2^2}$, $\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}$, $\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}$, ...

$\frac{1}{2}$, $\frac{1}{2}+\frac{1}{4}$, $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, ...

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, ...

We have been interested in $\lim_{n\to\infty} a_n$

i.e. $\lim_{n\to\infty} \frac{1}{2^n} = 0$ since GSeq $r = \frac{1}{2}$

For the next days we'll be interested in

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$$

$$= \lim_{n\to\infty} \frac{2^n - 1}{2^n} = 1$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2^k} = 1$$

We say that this seq of partial sums converges to $s = 1$ (the sum of the series)

We write $s = \lim_{n\to\infty} \sum_{k=1}^{n} a_k = \lim_{n\to\infty} s_n$

$$= \lim_{n\to\infty} \left(a_1 + a_2 + \cdots + a_n \right) = \sum_{k=1}^{\infty} a_k$$

Series Tests

① Geometric Series

Terms : $a_k = a \cdot r^k$, $a_n = a r^n$

Series $\displaystyle\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} a r^n$

$$s_n = a \cdot r^0 + a r^1 + a \cdot r^2 + \cdots + a r^n$$

$$\frac{s_n}{a} = 1 + r + r^2 + \cdots + r^n$$

$$r\frac{s_n}{a} = r + r^2 + r^3 + \cdots + r^{n+1}$$

$$r\frac{s_n}{a} - 1 \cdot \frac{s_n}{a} = r^{n+1} - 1$$

$$\frac{s_n}{a}(r-1) = r^{n+1} - 1$$

$$s_n = a\,\frac{r^{n+1}-1}{r-1}$$

$$\lim_{n\to\infty} s_n$$

$$= \lim_{n\to\infty} a\,\frac{r^{n+1}-1}{r-1}$$

$$= a\,\frac{0-1}{r-1}$$ since $|r|<1$

$$= a \cdot \frac{1}{1-r} = \frac{a}{1-r}$$