M 192 Lect #20 11-2-11

__Ex__ $\sum_{n=0}^{\infty} 4 \frac{1}{5^n}$ make it match $\sum_{n=0}^{\infty} a r^n$

conv if $|r|<1$ to $\frac{a}{1-r}$

$$= \sum_{n=0}^{\infty} 4 \left(\frac{1}{5}\right)^n = \lim_{n\to\infty} \sum_{k=0}^{n} 4\left(\frac{1}{5}\right)^k$$

$r = \left|\frac{1}{5}\right| < 1$

So the GS converges to

$$\frac{a}{1-r} = \frac{4}{1-\frac{1}{5}} = \frac{4}{\frac{5}{5}-\frac{1}{5}}$$

$$= \frac{4}{\frac{4}{5}} = \cancel{4}\cdot\frac{5}{\cancel{4}} = 5$$

$\sum_{n=0}^{\infty} 4 \frac{1}{5^n}$ conv to 5

sum of series is 5

$$\sum_{n=2}^{\infty} 7\,\frac{4^{n+2}}{5^{n-1}} \quad {\scriptstyle n+6} \leftarrow \text{what if} \qquad \sum_{n=0}^{\infty} a r^n$$

$$= \sum_{n=0}^{\infty} 7\,\frac{4^{n+4}}{5^{n+1}}$$

$$5^{n-4}$$

$$= \sum_{n=0}^{\infty} 7\,\frac{4^n \cdot 4^4}{5^n \cdot 5^1}$$

$$5^n \cdot 5^{-4}$$

$$= \sum_{n=0}^{\infty} \left(7\,\frac{4^4}{5^1}\right)\left(\frac{4}{5}\right)^n$$

$$5^{-4}$$

$$|r| = \left|\frac{4}{5}\right| = \frac{4}{5} < 1$$

So GS conv to

$$\frac{a}{1-r} = \frac{\frac{7\cdot 256}{5}}{1-\frac{4}{5}} = \frac{\frac{7\cdot 256}{5}}{\frac{1}{5}} = \frac{7\cdot 256}{\cancel{5}}\cdot\frac{\cancel{5}}{1}$$

$$= 1792.$$

So sum of series is 1792

(2) Partial Fractions Series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+3n} = \frac{1}{4} + \frac{1}{10} + \frac{1}{18} + \frac{1}{28} + \frac{1}{40}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+3n} = \sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

$$= \sum_{n=1}^{\infty} \frac{1/3}{n} + \frac{1/-3}{n+3}$$

$$= \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3}$$

$$= \frac{1}{3}\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{2} - \frac{1}{5} + \frac{1}{3} - \frac{1}{6} + \frac{1}{4} - \frac{1}{7} + \underbrace{\frac{1}{5} - \frac{1}{8}}^{n=5} + \cdots \right]$$

$$S_8 = \frac{1}{3}\left[\frac{1}{1} - \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{5}} + \frac{1}{3} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{6}} - \frac{1}{9} + \cancel{\frac{1}{7}} - \frac{1}{10} + \cancel{\frac{1}{8}} - \frac{1}{11} \right]$$

$$= \frac{1}{3}\left[1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{9} - \frac{1}{10} - \frac{1}{11} \right]$$

$$A_n = \frac{1}{3}\left[1+\frac{1}{2}+\frac{1}{3}-\frac{1}{n+1}-\frac{1}{n+2}-\frac{1}{n+3}\right]$$

$$\lim_{n\to\infty} A_n = \frac{1}{3}\lim_{n\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}-\cancel{\frac{1}{n+1}}^{\,0}-\cancel{\frac{1}{n+2}}^{\,0}-\cancel{\frac{1}{n+3}}^{\,0}\right]$$

$$= \frac{1}{3}\left[1+\frac{1}{2}+\frac{1}{3}\right] = \frac{1}{3}\left[\frac{6}{6}+\frac{3}{6}+\frac{2}{6}\right]$$

$$= \frac{1}{3}\cdot\frac{11}{6} = \frac{11}{18}$$

So Original series conv. to $\frac{11}{18}$

③ Harmonic Series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots\cdots$$

A series whose terms go to zero and yet partial sums div.

$$s_5 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$s_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$s_{16} = \frac{1}{1} + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{> \frac{2}{4}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{> \frac{4}{8}} + \underbrace{\frac{1}{9} + \cdots + \frac{1}{16}}_{> \frac{8}{16}} \quad \frac{16}{32}$$

$$1 \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$$

$$> 1.5 + 3\left(\tfrac{1}{2}\right) = 1 + 4\left(\tfrac{1}{2}\right)$$

$$s_{32} > 1.5 + 4\left(\tfrac{1}{2}\right) = 1 + 5\left(\tfrac{1}{2}\right)$$

$$s_{2^{10}} > 1 + 10\left(\tfrac{1}{2}\right)$$

$$s_{64} > 1 + 6\left(\tfrac{1}{2}\right)$$

$$s_{2^6} > 1 + 6\left(\tfrac{1}{2}\right)$$

$$\lim_{n\to\infty} s_n = \infty$$

So harmonic series div

④ Divergence Test (n^{th} term test)

$$\sum_{n=1}^{\infty} a_n$$

If $\lim_{n\to\infty} a_n \neq 0$, $\sum_{n=1}^{\infty} a_n$ diverge

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So what happens if $\lim_{n\to\infty} a_n = 0$

Can't say yet, must use another test

Ex $\sum_{n=1}^{\infty} \frac{1}{40} = \sum_{n=1}^{\infty} a_n$

$a_n = \frac{1}{40}$

$\lim_{n\to\infty} a_n = \frac{1}{40} \neq 0$

So $\sum a_n$ div

Ex 2 $\sum_{n=1}^{\infty} \frac{n^2}{n^2+5000n}$

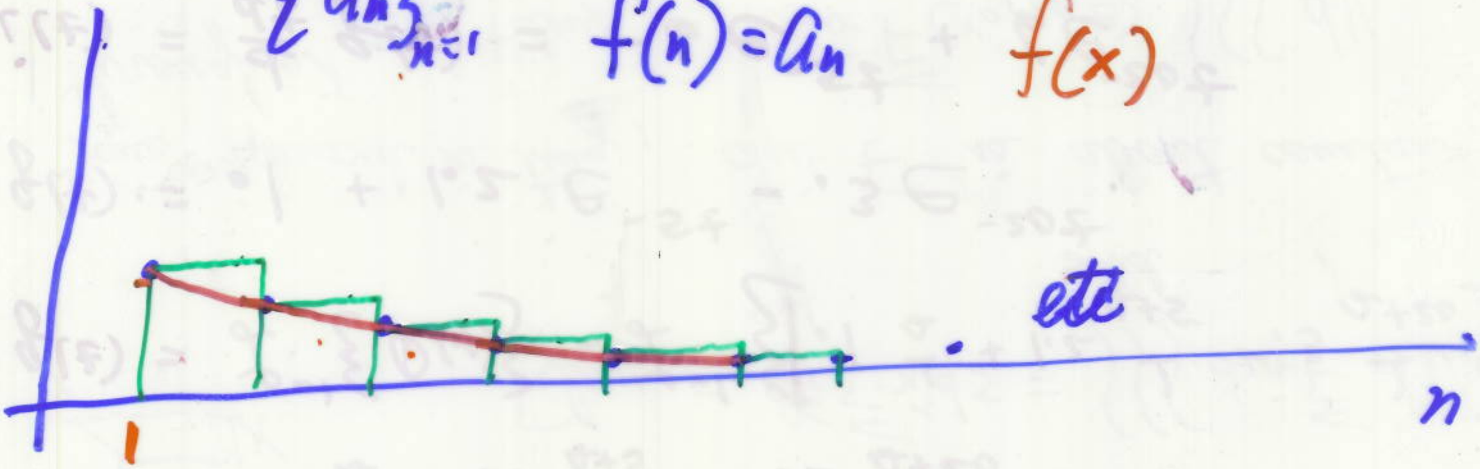
$\lim_{n\to\infty} a_n = \frac{n^2}{n^2+5000n} = 1 \neq 0$ So

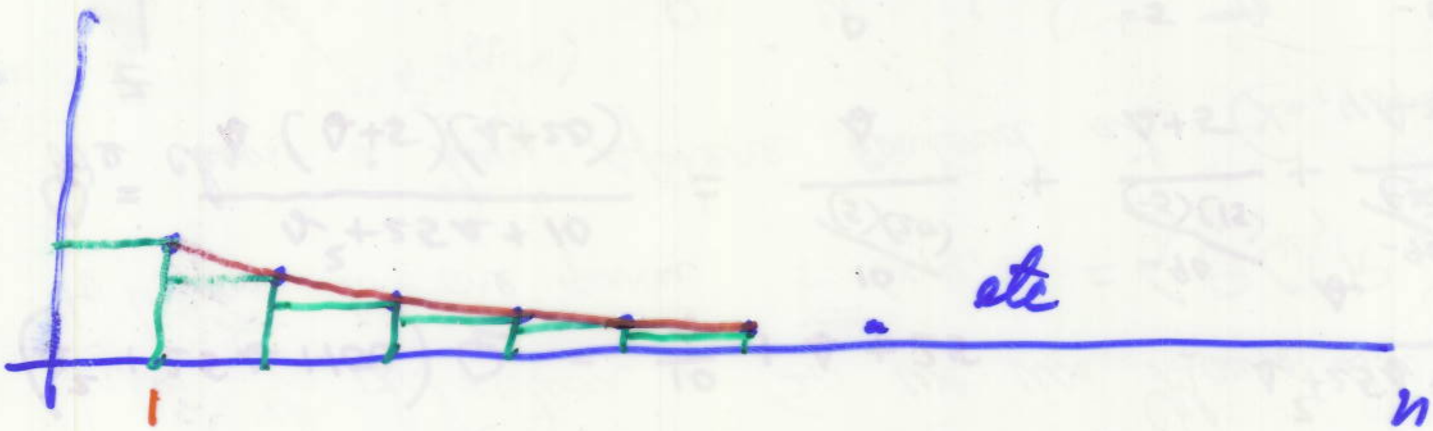
$\sum a_n$ div

⑤ Integral Test

(a connect the dots series test

$\{a_n\}_{n=1}^{\infty}$ $f(n) = a_n$ $f(x)$





$$\sum_{n=2}^{\infty} a_n \le \int_1^{\infty} f(x)\,dx \le \sum_{n=1}^{\infty} a_n$$