M 192 Lect #21 11-7-11

(5) Integral Test Example

Given $\sum a_n = \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ ← a_n pos, a_n decr

Create $f(x) = \frac{1}{x^2}$ ← (i.e. connect the dots)

f is pos, decr, easily integrable.

$$\int_1^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^t x^{-2}\,dx$$

$$= \lim_{t\to\infty} \left. \frac{x^{-1}}{-1} \right]_1^t = -\lim_{t\to\infty} \left. \frac{1}{x} \right]_1^t$$

$$= -\lim_{t\to\infty} \left(\frac{1}{t} - \frac{1}{1} \right) = +1$$

($\frac{1}{t} \to 0$ as $t \to \infty$)

Since improper integral conv.

Then the original series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

(5) Ex 2 Int Test

$$\sum a_n = \sum \frac{1}{n^{1.1}}$$

$$f(x) = \frac{1}{x^{1.1}}$$

$$= \int_1^\infty x^{-1.1}\,dx = \lim_{t\to\infty} \int_1^t x^{-1.1}\,dx$$

$$= \lim_{t\to\infty} \frac{x^{-.1}}{-.1}\Big]_1^t = \frac{-1}{.1} \lim_{t\to\infty} \left(\frac{1}{t^{.1}} - \frac{1}{1^{.1}}\right)$$

($t^{.1} \to \infty$, so $\frac{1}{t^{.1}} \to 0$)

$$= -10\,(-1) = 10$$

Since improper int converges
Then orig series also conv.

⑥ The p-series is $\sum_{n=1}^{\infty} \frac{1}{n^p}$

We conjecture that the

p-series { conv for $p > 1$
div for $p \leq 1$

Ex of ⑥ $\sum_{n=1}^{\infty} \frac{14}{\sqrt{n}} = 14 \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

This is a p series with $p = \frac{1}{2} < 1 \Rightarrow$
Orig series diverges

⑧ Limit Comparison Test

Given $\sum a_n$

Create $\sum b_n$ < similar to $\sum a_n$ / easier than $\sum a_n$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

If $0 < L < \infty$

then both series do the same i.e.

if $\sum b_n$ conv so does $\sum a_n$.

if $\sum b_n$ div so does $\sum a_n$.

⑧ Example

what if 7

Given $\sum a_n = \sum_{n=1}^{\infty} \frac{n^2+3}{n^3-5n+8}$ (4)

Create $\sum b_n = \sum \frac{n^2}{n^3} = \sum \frac{1}{n}$

Piece [1]: $\sum b_n = \sum \frac{1}{n}$ div by ③ harmonic series

Piece [2]: $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{7n^2+3}{4n^3-5n+8}}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{7n^3+3n}{4n^3-5n+8}$ correct data & use Calc!

$= \lim_{x\to\infty} \frac{7\frac{x^3}{x^3}+\frac{3x}{x^3}}{\frac{x^3}{x^3}-\frac{5x}{x^3}+\frac{8}{x^3}}$ $= \frac{7}{4} = L$

Since $L = 1$, $0 < L < \infty$, so both series do the same.

Since $\sum b_n$ div then $\sum a_n$ diverges

(10) Ratio Test | (11) Root Test

(These two tests work for series that converge or diverge really fast.)

For a series: $\sum a_n$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L \quad \text{or} \quad \lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

Then if $L < 1$ series conv absolutely

i.e. $\sum_{n=1}^{\infty} |a_n|$ conv

if $L \geq 1$ series diverges

(10) Example $\sum a_n = \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{4^n + n}$

Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} \frac{3^{n+1}}{4^{n+1}+n+1}}{(-1)^n \frac{3^n}{4^n+n}} \right|$$

$$= \lim_{n\to\infty} \frac{3^{n+1}}{4^{n+1}+n+1} \cdot \frac{4^n+n}{3^n}$$

$$= \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \cdot \lim_{n\to\infty} \frac{4^n+n}{4^{n+1}+n+1} \qquad \frac{3^{n+1}}{3^n} = 3$$

Connect the dots

$$= 3 \cdot \lim_{x\to\infty} \frac{4^x + x}{4^{x+1}+x+1}$$

$$\overset{H}{=} 3 \cdot \lim_{x\to\infty} \frac{4^x \cdot \ln 4 + 1}{4^{x+1} \cdot \ln 4 \cdot 1 + 1 + 0}$$

$$\overset{H}{=} 3 \cdot \lim_{x\to\infty} \frac{4^x \ln 4 \ln 4}{4^{x+1} \ln 4 \ln 4} = 3 \cdot \frac{1}{4} = \frac{3}{4} = L$$

$L = \frac{3}{4} < 1$

$\Rightarrow$

Series converges absolutely

Prob Set Hints

1. Word for Word

A series $\sum a_n$ conv

if the seq of partial sums s_n converge. $s_n = \sum_{k=1}^{n} a_k$.

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2. (a) (b)

sand.

geom

3. Series (1) geom.
(2) partial frac.

4.

a	lim comp	(8)	(g)	Alt ser (skip)	
b	alt ser. (skip)		(h)	Ratio	(10)
c	Div	(4)	(i)	Comp	(7) skip
d	Root	(11)	(j)	dist	(5)
e	Ratio	(10)	(k)	root	
f	Div	(4)	l	skip	
			m	skip	