M 192 Lect #22 11-9-11

Some little facts:

1. You can ignore the first few terms of a series in determining the conv/div of the series.

2. Given two series $\sum a_n$ $\sum b_n$

Suppose		Find
$\sum a_n$	$\sum b$	$\sum (a_n + b_n)$
C	C	C
C	D	D
D	C	D
D	D	Can't tell

$\sum a = \sum \frac{1}{n}$ $\sum b = \sum -\frac{1}{n}$

$\sum (a_n + b_n)$
$= \sum \left(\frac{1}{n} - \frac{1}{n}\right) = \sum 0$
$= 0$

Chap 12 - B §12.8 – 12.12

Power Series $\leftarrow x^n$ or $(x-a)^n$

$$\sum a_n = \sum c_n x^n \text{ or } \sum c_n (x-a)^n$$

A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

Our goal is to create lots and lots of power series that equal functions

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We know G.S. ①

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Let $a=1$ chang r to x and swap sides

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{conv for } |x| < 1$$

source = tree top

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \text{conv for } |x| < 1$$

Replace x by $3x$

$$\frac{1}{1-(3x)} = \sum_{n=0}^{\infty} (3x)^n \qquad \text{conv for } |3x| < 1$$

$$\frac{1}{1-3x} = \sum_{n=0}^{\infty} 3^n x^n \qquad |x| < \frac{1}{3}$$

Replace x by $-x$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-x)^n \qquad |-x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \qquad |x| < 1$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \qquad |x| < 1$$

Replace x by $x-1$

$$\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n \qquad |x-1| < 1$$

source

p4

$$\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n \qquad |x-1| < 1$$

← radius of conv

$-1 < x-1 < 1.$

$0 < x < 2$

The interval of conv is $(0,2)$

Integrate each term

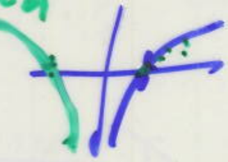
$$\int \frac{1}{x}\,dx = \int \sum_{n=0}^{\infty} (-1)^n (x-1)^n\,dx \qquad |x-1|<1$$

$$\ln|x| = \sum_{n=0}^{\infty} \int (-1)^n (x-1)^n\,dx$$

$u = x-1$
$du = dx$

$$\ln|x| = \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1} + C$$

$$\ln|x| = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x-1)^n + C$$



$\ln(1) = 0$

$$\ln(1) = 0 = \sum_{n=1}^{\infty} 0 + C \Rightarrow C = 0$$

target

$$\ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x-1)^n \qquad |x-1| < 1$$

p5

Given a power series, we wish to find its "radius of convergence and interval of convergence"

$$\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{4^{n+2}} (x-5)^n = \sum a_n$$

Use ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(-1)^{n+1} \frac{3^{n+1}}{4^{n+3}} (x-5)^{n+1}}{(-1)^n \frac{3^n}{4^{n+2}} (x-5)^n} \right|$$

$$\boxed{\lim_{n\to\infty} = \frac{3^{n+1}}{4^{n+3}} |x-5|^{n+1} \frac{4^{n+2}}{3^n |x-5|^n}}$$

$$= \lim \frac{3^{n+1}}{3^n} \cdot \lim \frac{4^{n+2}}{4^{n+3}} \cdot \lim \frac{|x-5|^{n+1}}{|x-5|^n}$$

Source Tree Top

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^{n} \qquad |x|<1$$

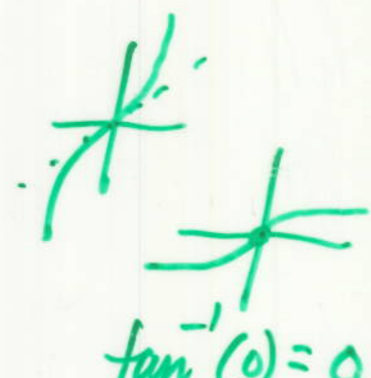
Replace x by $-x^2$

$$\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n \qquad \text{for } |-x^2|<1$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \qquad \text{for } |x|<1$$

$$\int \frac{1}{1+x^2}\,dx = \sum_{n=0}^{\infty} (-1)^n \int x^{2n}\,dx \qquad |x|<1$$

$$\tan^{-1}x + C_1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} + C_2$$



$$0 + C = \sum 0 = 0$$

$$\tan^{-1}(\underset{\substack{\uparrow \\ x}}{0}) = 0$$

target

$$\tan^{-1}x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \qquad |x|<1$$