M 192 Lect # 24 11-16-11

Let's form the Taylor polynomial $P_4(x)$ and Taylor series for $f(x) = e^x$ expanded about $x = a = 0$

n	$f^{(n)}(x)$	$f^{(n)}(a)$
0	e^x	$e^0 = 1$
1	e^x	$e^0 = 1$
2	e^x	$e^0 = 1$
3	e^x	$e^0 = 1$
4	e^x	$e^0 = 1$
5	e^x	e^z

contributes to the remainder

$$f(x) \approx P_4(x) = \frac{f^{(0)}(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2$$

$$+ \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= \frac{1}{1}\cdot 1 + \frac{1}{1}x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$$

$$P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$$

Taylor series is

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!}x^n = e^x$$

rad of conv next time

Now we have a new tree top and can make lots of new series.

New tree top

Source

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$ Conv for all x

want ① replace x by $5x$

$$e^{5x} = \sum_{n=0}^{\infty} \frac{(5x)^n}{n!} = \sum_{n=0}^{\infty} \frac{5^n}{n!} x^n$$ all x

form ② mpy series by x

$$xe^x = \sum_{n=0}^{\infty} \frac{1}{n!} x x^n$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} x^{n+1}$$ all x