

Name \_\_\_\_\_

Note: Evaluate some of the definite integrals by hand and some by calculator.

1. Find the area bounded by  $x - y^2 = 0$  and  $x - y = 6$ .

a. Write the integral using  $dx$ .

b. Write the integral using  $dy$ .

2. Describe and find the volume of the solid generated by revolving one hump of the sine curve

a. about the  $x$ -axis;

b. about the  $y$ -axis.



5. A spring exerts a force of 5 lbs when stretched to a length of 8 inches beyond its natural length .
- Find the spring constant and the force function  $f(x)$  where  $x$  is the distance stretched beyond its natural length.
  - Find the amount of work required to stretch the spring 12 inches beyond its natural length.
  - Find the amount of work required to stretch the spring from 20 inches to 32 inches beyond its natural length
  - Why are these two answers different when the stretch was the same in both?

6. To reduce the risk of contamination of the water table, the EPA (Environmental Protection Agency) requires removal of all under ground oil storage tanks. A cylindrical oil tank of radius 12 ft and height 8 feet standing on its end is half filled with oil having density 50 pounds per cubic foot. The flat bottom of the tank is 14 feet below ground level. The top hatch of the oil tanker where the oil is to be pumped is 10 feet above the ground. How much work is required to pump the rest of the oil from the tank to the tanker.

7. State the mean value theorem for integrals. (You should copy this verbatim from the text.)
8. Find the value of  $c$  whose existence is guaranteed by the Mean Value Theorem for Integrals and the average value of  $f$  for the function,  $f(x) = x^2 - 4x + 6$ , on the interval  $[1,7]$ . Draw a graph showing the function, the value  $c$  and the average value of  $f$ .

9. A dangling 100 foot cable weighing 300 lbs is being wound up. A 700 lb weight is attached to the end.
- a. How much work is done in winding up the entire cable and weight?

b. How much work is done in winding up just half of the cable and the weight.

10. For the inverse cosine function,  $\cos^{-1}$ , do all of the following.

a. Give the definition, domain & range.      b. Evaluate  $\cos^{-1}(-\frac{1}{2})$

c. Draw the graph.

d. Find the derivative of  $y = \cos^{-1}(3x)$ .

e. Evaluate this integral.

f. Derive the formula for the derivative of

$$\int_0^{1/2} \frac{1}{\sqrt{9-x^2}} dx$$

$$y = \cos^{-1}(x).$$

11. Determine whether l'Hopital's rule applies to the following and find the limit in each.

a. 
$$\lim_{x \rightarrow 0} \frac{x + \tan(x)}{\sin(x)}$$

b. 
$$\lim_{x \rightarrow \infty} \frac{e^x}{x^4}$$

c. 
$$\lim_{x \rightarrow \infty} \frac{e^x}{x^n}$$
 where  $n$  is any fixed positive constant.

Draw a conclusion about the growth rate of exponential functions versus power functions.

d. 
$$\lim_{x \rightarrow 0^+} x^2 \ln(x)$$



e.  $\lim_{x \rightarrow 1} \left( \frac{1}{\ln(x)} - \frac{1}{x-1} \right)$

f.  $\lim_{x \rightarrow 0^+} (\sin(x))^{\tan(x)}$

g.  $\lim_{x \rightarrow 0} (\csc(x))^{\tan(x)}$

h.  $\lim_{x \rightarrow 0^+} (\cos(x))^{\frac{1}{x}}$