Math 192 Problem Set #3 for Chapters 9&10 C.S.Davis/M.K.Glasby

Name______________________________

1. Bacteria is growing in a culture at a rate proportional to the population of bacteria present at any time. The researcher estimates that there are 40 thousand bacteria initially and 120 thousand after 5 hours.
   a. Give the initial value problem (differential equation & initial conditions) for the number of bacteria B present at any time t.
   b. Solve the initial value problem.
   c. How many bacteria will be present after 24 hours?
   d. When will the bacteria reach one million?

2. A cup of coffee is 180°F when poured. After sitting out in the 70°F cafeteria for 10 minutes, its temperature is 120°F. Use Newton's law of cooling for the following.
   a. Give the initial value problem (differential equation & initial conditions) for the temperature T at any time t.
   b. Solve the initial value problem.
   c. What will the temperature be one hour after it is poured?
   d. When will the Temperature reach 75°F?

3. Now we attempt as many of the "dozen" aspects as we can on a parametrically defined function. Consider the function defined parametrically by the equations below.

   $x = f(t) = t^3 + 3$ and $y = g(t) = t^2 - 4t$ for t in [-2,4].

   Suppose x and y represent the position (x,y) of an object measured in feet and t represents time in seconds. Do all of the following.

   a. Find the position at time t=1.
   b. Eliminate the parameter to write the equation in rectangular coordinates.
   c. Graph the function using its rectangular form.
   d. Graph the function using its parametric form. Label where the object is at t = -2, -1, 0, 1, 2, 3 and 4.
   e. Find $\lim_{t \to \infty} f(t)$
   f. Find dx/dt, dy/dt, dy/dx, and d^2y/dx^2.
   g. Find the time t and place (x,y) where the graph has a horizontal tangent.
   h. Find the time t and place (x,y) where the graph has a vertical tangent.
   i. Find the slope of the tangent line at t = 1.
   j. Give the equation of the tangent line at t = 1 in rectangular form.
   k. Give the equation of the tangent line at t = 1 in parametric form.
   l. Find the area between the x-axis and the part of the curve below the x-axis. Do the integral by hand.
   m. Find the speed of the object at time t = 1.
      speed = $((dx/dt)^2 + (dy/dt)^2)^{1/2}$
   n. Set up an integral for the length of the entire curve over its domain. Evaluate the resulting integral using the TI.
   o. Revolve the curve about the x-axis and set up the integral for the surface area for that surface of revolution. Evaluate the integral using the TI.

4. Now we attempt as many of the "dozen" aspects as we can on polar equations and functions. Consider the polar functions defined by the equations below.

$r = 3\cos(2\theta)$ (the rose) and $r = 1$ (the circle)

a. Graph the rose and the circle. (In the TI86, you may use the pol mode.)
b. Convert the rose to an equation in rectangular coordinates.
c. Find dy/dx for the rose.
d. Find the smallest positive angle θ for which the rose has a horizontal tangent.
e. Find the smallest positive angle θ for which the rose has a vertical tangent.
f. Find the slope of the tangent line at $\theta=\pi/6$.
g. Give the equation of the tangent line at $\theta=\pi/6$ in rectangular form.
h. Convert the equation of the tangent line at $\theta=\pi/6$ from rectangular form to polar form.
i. Give the equation of the tangent line at $\theta=\pi/6$ in parametric form.
j. Find the area inside of one petal of the rose and outside of the circle.
k. Set up an integral for the arc length of the entire boundary of the rose. Evaluate the resulting integral using the TI.
l. Revolve one petal about the x-axis and set up the integral for the surface area for that surface of revolution. Evaluate the integral using the TI.