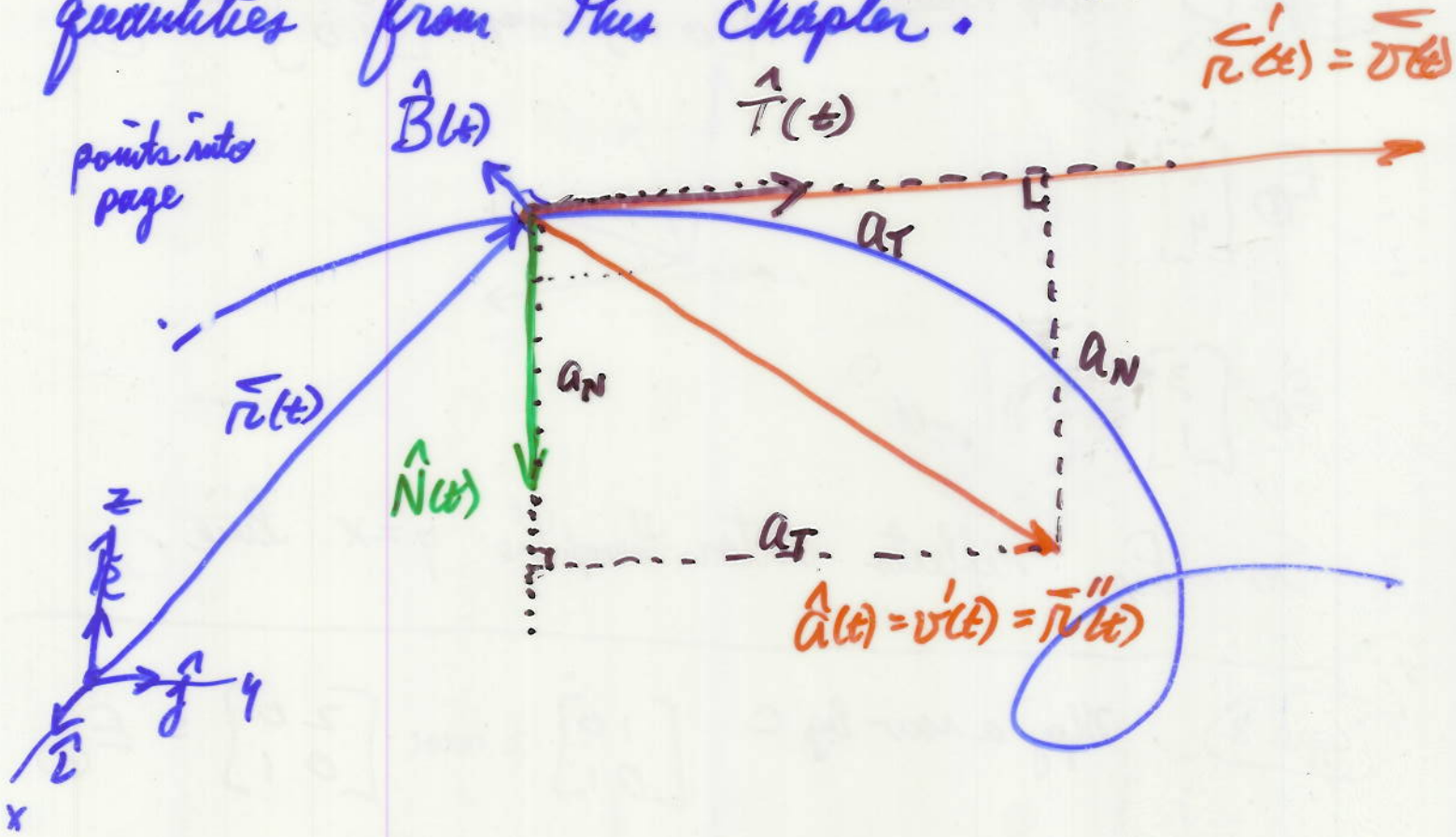
M201 Lect #9 E 2-15-12

Let's draw a diagram with "all" of the quantities from this chapter.



$\hat{T}(t)$ points where the curve is going

$N(t)$ point where the curve is tending to go

$B(t)$ points $\perp$ to both of the above

Chapter 15 Functions of Several Variables (Surfaces)

$z = f(x,y)$ or $F(x,y,z) = 0$

$z = x^2 + y^2$ $\qquad$ $x^2 + y^2 + z^2 - 1 = 0$

We wish to begin with a fcn of 2 var in algebraic form and draw its graph (surface) We will use the method of traces, time only horizontal traces, this time requiring that all the zs be equally spaced. The method is called the method of level curves.

Ex Use method of level curves to graph

$$z = f(x,y) = x^2 + y^2$$

$z = 0 = x^2 + y^2 = 0^2$

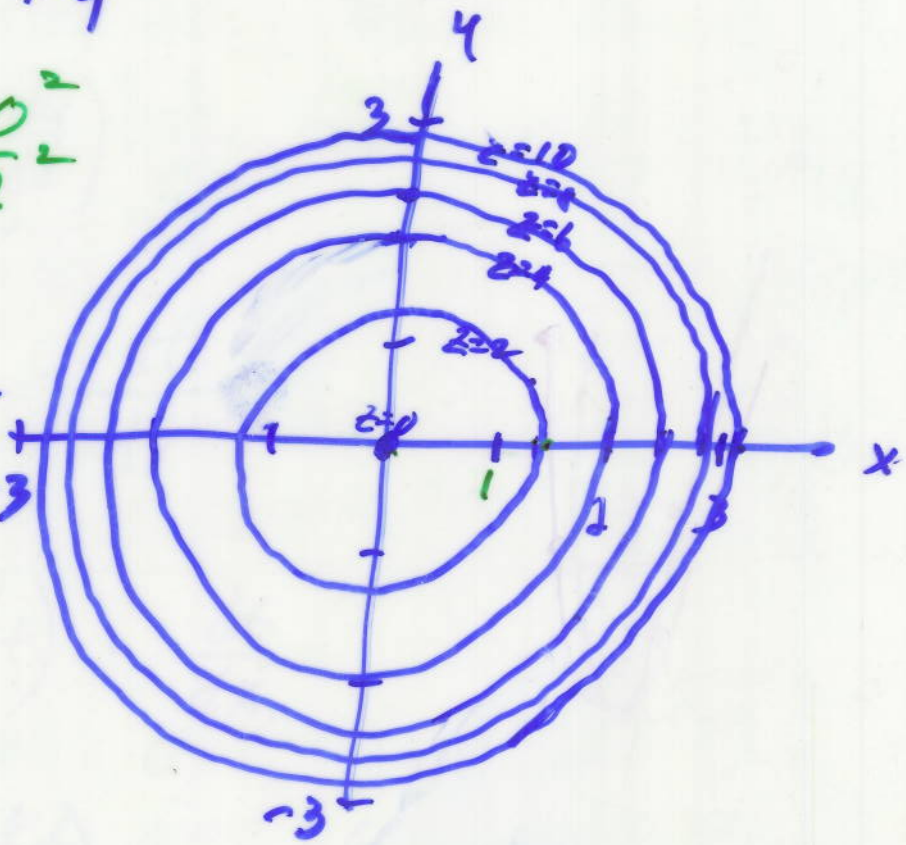
$z = 2 = x^2 + y^2 = \sqrt{2}^2$

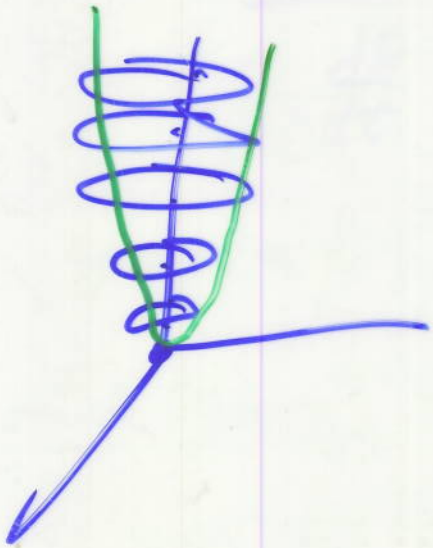
$z = 4 = x^2 + y^2$

$z = 6 = x^2 + y^2 = \sqrt{6}^2$

$z = 8 = x^2 + y^2 = \sqrt{8}^2$

$z = 10$



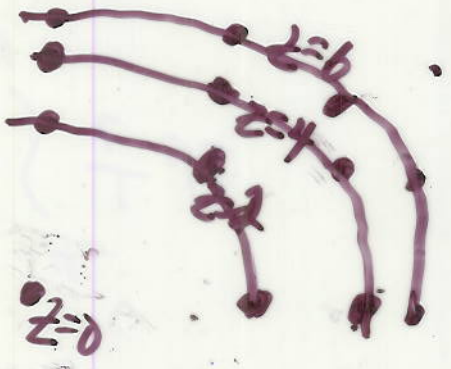


So the closer the level curves, the steeper the surface

Table for func of 2 var $z = x^2 + y^2$

y \ x	0	1	2	3
3	9	10	13	18
2	4	5	8	13
1	1	2	5	10
0	0	1	4	9

← z's


z=b
z=z
z=1
z=0

Limits

In Case I

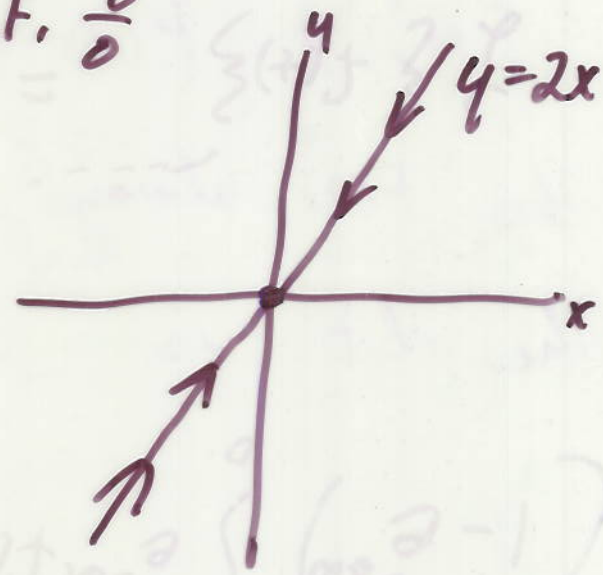
In Case III

Easy One

$$\lim_{(x,y)\to(2,3)} \frac{3x-4}{5x+4} = \frac{3}{14}$$

I.F. $\frac{0}{0}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} \quad (x^2 \to 0,\ y^2 \to 0)$$



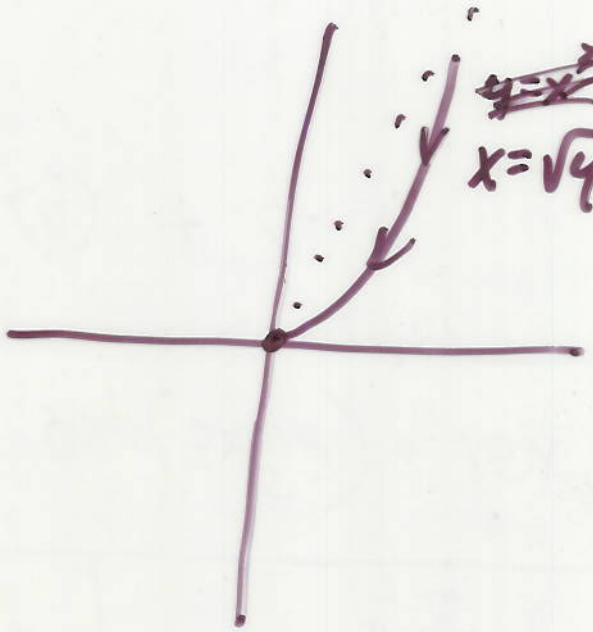
along $y = 2x$

$$= \lim_{x\to 0} \frac{x^2-(2x)^2}{x^2+(2x)^2}$$

$$= \lim_{x\to 0} \frac{-3x^2}{5x^2} = -\frac{3}{5}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

along $x=\sqrt{y}$



$$= \lim_{y\to 0} \frac{\sqrt{y}^2-y^2}{\sqrt{y}^2+y^2}$$

$$= \lim_{y\to 0} \frac{y-y^2}{y+y^2} = \lim_{y\to 0} \frac{y(1-y)}{y(1+y)} = 1$$

Since $-\frac{3}{5} \neq 1$

the limit DNE (does not exist)