Name ________________________________

There are but two non-trivial vector valued functions for which we can easily calculate the interesting quantities in all aspects of space curve analysis. One is the subject of this problem set and the other is the circular helix. In order to succeed for this problem set, we must recall some properties of the hyperbolic functions.

$\sinh t = \dfrac{e^t - e^{-t}}{2}$ $\cosh t = \dfrac{e^t + e^{-t}}{2}$ sech t = 1/cosh t tanh t = sinh t / cosh t

$\dfrac{d}{dt}\cosh t = \sinh t$ $\dfrac{d}{dt}\sinh t = \cosh t$ $\cosh^2 t - \sinh^2 t = 1$ $\text{sech}^2 t + \tanh^2 t = 1$

1. Consider the vector function **r** = **r**(t) = <5, 20 t, 20 cosh t > defined on the interval [-2,3], where t is time and **r**(t) is position.

   a. Give $\lim_{t \to 2}$ **r**(t)

   b. Give and evaluate the integral of **r**(t) over the whole domain.

   c. Give the integral for the arc length of the whole curve. Since this is one of the very few arc length integrals we can evaluate by hand, do so and then check it with your calculator.

d. Suppose $t = g(u) = 6u^3 + 4u^2$. Use the chain rule to find $d\mathbf{r}/du$.

e. Give the velocity vector, $\mathbf{v}(t) = \mathbf{r}'(t)$ in terms of the **i, j, k** basis.

f. Give the speed.

g. Give the unit tangent vector, $\mathbf{T}(t)$.

h. Give the unit normal vector, $\mathbf{N}(t)$.

i. Give the unit binormal vector, $\mathbf{B}(t)$.

j. Give the parametric equations of the tangent line to the curve at the point where $t = 2$.

k. Give the definition of curvature, K, and then find the curvature using the definition.

l. Find the curvature using the formula involving a cross product.

m. Find the acceleration, **a**(t), in terms of **i, j, k** basis.

n. Find the tangential component of acceleration, a_T.

o. Find the normal component of acceleration, a_N.

p. Give the velocity, **v**(t), in terms of the **T, N, B** basis. (Some coefficients may be zero; include them as zero for completeness.)

q. Give the acceleration, **a**(t), in terms of **T, N, B** basis.

r. Use the chain rule to find d**r**/ds.

s. Draw a large diagram depicting **r**, **r'**, **r''**, **T**, **N**, **B**, **v**, **a**, a_T and a_N for the curve given by **r**(t) at the point where t = 2. Tell the significance of each quantity in a sentence.

2. Give the trivial parameterization of the rectangular equation $y = 3x^2 + e^x$.

3. Give parametric equations and a vector equation for a right circular helix having radius 8 and lying about the vertical line passing through the point (2,3,4). Give the domain for the parameter t so that the spring (helix) will have 10 coils.

4. Give the parametric form of the equations of the curve of intersection of the plane, $x + 2y - 3z = 4$, and the surface, $2x + y^2 + 4y - 8z = 8$.