Math 201 Problem Set #3 on Chapter 14 C.S.Davis

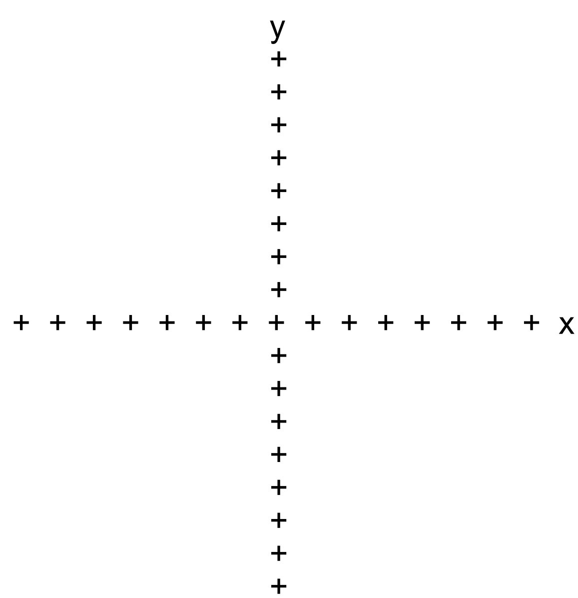
Name ________________________________

1. Determine as best you can whether this limit exists and if so, find it. Draw your conclusion after testing with a line, a parabola, a cubic and a quartic that pass through the origin.

$$\lim_{(x,y)\to(0,0)} \frac{12x^4 y}{x^8 + y^2}$$

2. a. Graph several of the level curves of the function below on the axes below.

   $z = f(x,y) = y^2 - x$



   b. Now describe and graph the surface $z = f(x,y)$.

3. Graph the domain of the function $f(x,y) = \begin{cases} \sqrt{x^2+y^2-9} & \text{for } 9 \le x^2+y^2 \\ 2 & \text{for } 4 \le x^2+y^2 < 9 \end{cases}$

4. Determine whether f(x,y) in problem 3 above is continuous over its entire domain and give the reason for your conclusion.

5. For the function $z = f(x,y) = e^{3x} \cdot y^5 + \sin(x) - \cosh(3y) + 5$

   a. Give the partial derivative of z with respect to y.

   b. Give $f_x(x,y)$

   c. Give the second partial of z with respect to y second.

   d. Give $f_{xy}(x,y)$

5. Continued
   e. Suppose that the x and y in the function f are given by

      $x = 5t^3 - 2$ and $y = 2t^4 + 3t$.

      Give the derivative of z with respect to t using the chain rule. Do not substitute for x and y.

   f. If $x = 3\cos(t)*\ln(s)$ and $y = t^4 + s^3$, give the partial of z with respect to t. Do not substitute for x and y.

6. Use implicit differentiation to find $\partial z/\partial x$ in the equation $x^3z + 5xy + 3y\sin(z) = 2z^2$.

7. For a surface $z = g(x,y)$, explain the distinction between Δz and dz. Give formulas for each and a geometric explanation.

8. Work this exercise about wave heights from Stewart: Section 14.3 #4.

9. Work this exercise about a contour map from Stewart: Section 14.3 #10.

10. For the function $z = f(x,y) = x^3 y^5 + 2x - 7y$

a. Give **grad** f at (x,y).

b. Give a vector normal to the surface at the point on the surface where x=2 and y=1.

c. Give the equation of the plane tangent to $z = f(x,y)$ at the point on the surface where x=2 and y=1.

d. Give a vector (actually a vector function) which gives the normal to the level curve $7 = x^3 y^5 + 2x - 7y$ at each (x,y) on the level curve.

e. Give the directional derivative $D_{\mathbf{u}}f$ in the direction $<12,-5>$at the point where x=2 and y=1.

f. Give the direction (in the xy-plane) in which the surface (and thus the tangent plane to the surface) at (2,1) is steepest.

g. Give the steepest slope of the tangent plane at (2,1).

11. For the function $z = f(x,y) = 12xy + x^3 - y^2$, locate all relative maxima, relative minima and saddle points. Use the discriminant test to support your conclusion.

12. For the function $f(x,y) = 5 + 4x - 2x^2 + 3y - y^2$ defined on the triangular region D bounded by the lines $y=-x$, $y=x$ and $y=2$, find the absolute maximum and absolute minimum values and the points at which they occur. Make a table that lists all critical values inside D, on the boundary of D and the corners of D.

13. Use the method of Lagrange Multipliers to find the point on the plane $6x + 4y - 2z = -58$ that is closest to the point $(2,-1,5)$. (Hint: Minimize the square of the distance from $(2,-1,5)$ to a typical point (x,y,z) on the plane.)