Name________________________________

1. Use the Riemann sum to estimate the volume of the solid that lies below the surface $z = 5x^2 + 3y^3$ and above the rectangle $R = \{(x, y) \mid 0 \le x \le 12, 1 \le y \le 5\}$.
Estimate using the Mid-point Rule with m = 3 and n = 2. Make a two dimensional table as part of your solution process.

2. Use the following table to estimate $\iint_R f(x,y)\, dA$ over R = [1, 7] x [2, 6].

The x values are in the bottom row, the y values are in the left column and the values of the function are in the remaining body of the table. Use the left endpoint rule, i.e., take the sample points to be the lower left corners.

6	9	14	22	27	30
5	7	12	18	22	26
4	5	10	14	16	20
3	4	8	11	13	15
2	3	6	8	9	12
1	3	4	5	6	7
0	2	2	3	4	5
↑ x→ y	1	3	5	7	9

For the rest of the problem set, "Set up only." means to set up the integral ready to integrate without further information or substitutions, but <u>not</u> to evaluate the integral (unless you just want the practice). "Evaluate" means to set up and evaluate the integral to get a number for the answer.

3. Find the volume of the following solids.
   a. Under the paraboloid $z = 2x^2 + 5y^2$ and above the rectangle [2, 4] x [-3, 1]. (Set up only.)

   b. Bounded by the surfaces $z = x + e^y$, $z = 0$, $y = x^2+1$ and $x + y = 3$. (Set up only.)

   c. Bounded by the plane $z = 3$ and the paraboloid $z = 53 - 2x^2 - 2y^2$. Convert to polar coordinates before integrating. (Evaluate.)

4. Evaluate the integral by sketching the region and reversing the order of integration. (Evaluate.)

$$\int_0^3 \int_{y^2}^9 \cos(x^{3/2}) \; dx\, dy$$

5. A lamina (flat sheet) lying in the first quadrant is bounded by $y=0$, $y=2$, $x=0$ and the right branch of the hyperbola $x^2-y^2 = 1$.

   a. Find the center of mass of the lamina if the density at any point is the square of its distance from the x-axis. (Set up only in rectangular coordinates.)

   b. Find the moments of inertia I_y and I_0 for the lamina. (Set up only in rectangular coordinates.)

6. Find the area of the surface of the part of the sphere $x^2 + y^2 + z^2 = 25$ above $z = 4$. (Set up only in polar coordinates.)

7. Consider the tetrahedron bounded by the plane $4x + 8y + z = 16$ and the 3 coordinate planes.

   a. Give the iterated integrals for the volume of the tetrahedron which uses the dz dx dy order of integration. (Set up only.)

   b. Give the iterated integrals for the volume of the tetrahedron which uses the dy dz dx order of integration. (Set up only.)

8. Use a triple integral in rectangular coordinates to find the volume of the wedge lying in the first octant bounded by the cylinder $y^2 + z^2 = 1$ and the planes $y = x$ and $x = 0$. (Set up only using the order dz dy dx.)

9. Use a triple integral in cylindrical coordinates to find the volume of the solid in the first octant inside of the sphere $x^2 + y^2 + z^2 = 4$ and inside the cylinder $x^2 + y^2 = 2x$. (Set up only.)

10. Use a triple integral in spherical coordinates to find the volume of the solid hemisphere bounded above and to the left by the sphere $x^2 + y^2 + z^2 = 16$ and bounded below and to the right by the plane $z = \sqrt{3}y$. Set up the triple integral in spherical coordinates here, then calculate the value of the integral with Maple using the commands, with(student), Tripleint(...), value(%) and evalf(%) to show that the answer gives the same volume as the formula for the volume of half a sphere. Bring the print-out to class. (For extra credit, you can also write the triple integrals for the volume in rectangular and cylindrical coordinates and have Maple check them as well.)

11. Convert the equation $x^2 + 5y^2 + 2x - 4z = 10$ from rectangular coordinates to cylindrical and spherical coordinates.

12. Convert the equation $\rho + 2\cos(\varphi) - 5\sin(\varphi)\cos(\theta) = 0$ from spherical coordinates to rectangular and cylindrical coordinates.

13. Describe, sketch and name the surfaces determined by these simple equations in the three dimensional coordinate systems of which they are a part.

a. $y = 1$

b. $r = 2$

c. $\varphi = 3$

d. $\theta = 4$

e. $\rho = 5$