Name ____________________________________

1. Sketch a sufficient number of vectors to show the following force fields.
   a. **F** = < -x, y >   b. **F** = < -y, x >

2. Let **F** = < $x^2 + y^2$, -2xy > be a force field. Evaluate the line integral $\int_C F \cdot dr$ to find the work done in moving a particle from the point (0, 0) to (2, 12) by way of the following two paths.
   a. Along the two line segments (0, 0) -> (2, 0) and (2,0) -> (2, 12)
   b. Along the curve $y = 3x^2$.

   Conclude whether this line integral is independent of the path.

3. a. Use a test to determine whether the vector field **F**(x,y,z) = < $2xy\ln(z) + 4x^3\sin(z)$, $x^2\ln(z) + 3y^2$, $x^2y/z + x^4\cos(z)$ > is a conservative vector field, and thus whether the line integral in part (b) is path independent.

b. If this line integral is path independent, find a potential function and evaluate the integral using the potential function. If it is not, use a straight line segment from beginning to end to evaluate it.

$$\int_{(3,2,1)}^{(2,1,4)} (2xy\ln(z) + 4x^3\sin(z))dx + (x^2\ln(z) + 3y^2)dy + (x^2y/z + x^4\cos(z))dz$$

C

4. Use Green's QP theorem to evaluate the circulation integral $\oint_C x^2y^2dx + (x^2 - y^2)dy$ where C is the square with vertices (0, 0), (6, 0), (6, 6), and (0, 6).

5. Illustrate (verify) Green's QP theorem for $\int_C xydx + (x + y)dy$ where C is the unit circle $x^2 + y^2 = 1$.

left side =

right side =

6. Set up the following flux integrals (surface integrals).

a. **F** = < x, y, z > S is the upper half of the sphere $x^2 + y^2 + z^2 = 36$.

b. **F** = < x, y, z > S is the first octant portion of the plane $z = -2x - 4y + 4$.

7. Use Gauss's divergence theorem to find the value of the flux of the vector field **F** below through the closed surface S where **F =** $\langle y \sin x,\ y^2 z,\ x + 3z \rangle$ and the surface S is the cube bounded by $x = \pm 1,\ y = \pm 1,\ z = \pm 1$.

8. Illustrate (verify) Gauss's divergence theorem with the force function

F(x, y, z) = $< 2x, -2y, z^2 >$

where the surface S is the cylinder $x^2 + y^2 = 64$

and the planes z=0 and z=8.

left side =

right side =

9. Illustrate (verify) Stoke's curl theorem with the force function **F**(x, y, z) = <3z,4x,2y> where the surface S is $z = 9 - x^2 - y^2$, $z \geq 0$.

left side =

right side =

10. Use the given transformation to evaluate the integral $\iint_R (x+y)\,dA$

where R is the square with vertices (0,0), (2,3), (5,1) and (3,-2).
The transformation is x = 2u+3v, y=3u-2v.