Name ______________________________

1. Perform the matrix arithmetic for these matrices. $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ -2 & -1 \end{bmatrix}$

a. AB

b. BA

c. $4A + 3B^T$

d. Does A have an inverse? Give reasons for your answer.

2. Find the solution of the system below by the Gauss-Jordan method used on matrices. Specify the row operation you are using at each step. Check your solution by substituting it into each of the original equations.

$$\begin{aligned} 2x \qquad\quad + 6z &= 28 \\ -x + 2y - 13z &= -56 \\ -4y + 23z &= 96 \end{aligned}$$

Solution: ______________________________

Check:

3. Prove the following result using the properties of real numbers and the definition of matrix addition.

For any m by n matrix A and any n by p matrices B and C,

$$A(B + C) = AB + AC$$

4. A local merchant has two pet stores where he sells ordinary and exotic dogs and cats. The matrices below give the inventory and annual cost of upkeep of each animal kept in stock.

Inventory of Store A

$$\begin{array}{c|c} \ldots & \begin{array}{cc} \mathit{Dogs} & \mathit{Cats} \end{array} \\ \begin{array}{c} \mathit{Ord} \\ \mathit{Exo} \end{array} & \begin{bmatrix} 20 & 10 \\ 10 & 30 \end{bmatrix} \end{array}$$

Inventory of Store B

$$\begin{array}{c|c} \ldots & \begin{array}{cc} \mathit{Dogs} & \mathit{Cats} \end{array} \\ \begin{array}{c} \mathit{Ord} \\ \mathit{Exo} \end{array} & \begin{bmatrix} 10 & 30 \\ 40 & 50 \end{bmatrix} \end{array}$$

Upkeep Costs

$$\begin{array}{c|c} \ldots & \mathit{Dollars} \\ \begin{array}{c} \mathit{Dogs} \\ \mathit{Cats} \end{array} & \begin{bmatrix} 300 \\ 200 \end{bmatrix} \end{array}$$

Use a matrix sum, product and/or scalar product to supply the following.

a. Give a single matrix representing the total inventory.

b. Give a single matrix for the total upkeep cost for ordinary and exotic animals.

c. Suppose the merchant decides to triple the inventory in Store A and double the inventory in Store B. Give a single matrix for the total inventory.

5. a. Calculate the inverse of A using row operations. Specify the row operation you are using at each step.

$$A = \begin{bmatrix} 1 & 6 \\ 2 & 11 \end{bmatrix}$$

b. Find the inverse of A using the short-cut, i.e., using a determinant in the denominator.

c. Use the inverse of A to solve the matrix equation A **x** = **b**: $\begin{bmatrix} 1 & 6 \\ 2 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$

d. Write the column vector **b**$=\begin{bmatrix} 6 \\ 9 \end{bmatrix}$ as a particular linear combination of the columns of A.

e. Find $(A^T)^{-1}$.

6. Consider the matrix $A = \begin{bmatrix} 2 & 8 \\ 3 & 2 \end{bmatrix}$

a. Write A as a product of elementary matrices.

b. Multiply some elementary matrices to check your answer.

c. Writ e A^{-1} as a product of elementary matrices.

d. Multiply the elementary matrices to obtain A^{-1} as a single matrix.

e. In general, which elementary matrices are invertible?

7. Consider the system of linear equations $\left\{\begin{array}{c} x+2y-2z=1 \\ 2y+3z=3 \\ 4x-2y-3z=9 \end{array}\right\}$.

a. Write the system as a matrix equation of the form A $\mathbf{x}$ = $\mathbf{b}$.

b. Find an LU factorization of A, the coefficient matrix.

c. Solve for $\mathbf{y}$ in the equation L$\mathbf{y}$ = $\mathbf{b}$ using forward substitution.

d. Solve for $\mathbf{x}$ in the equation U $\mathbf{x}$ = $\mathbf{y}$ using back substitution.

8. A medical researcher is studying the spread of a virus in a population of 1000 laboratory mice. During any given week there is an 80% probability that an infected mouse will overcome the virus, and during the same week there is a 10% probability that a noninfected mouse will become infected. Three hundred mice are currently infected with the virus. How many will be infected next week? In 2 weeks? In 10 weeks? Give the “steady state” of infected mice.

9. A message was encoded using the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. The encoded message is transmitted and received as $\begin{bmatrix} 12 & 19 \\ 41 & 62 \\ 2 & 3 \\ 44 & 75 \\ 25 & 50 \end{bmatrix}$. Decode the message.

10. An industrial system of two industries, coal and steel, has the following inputs:

To produce one dollar’s worth of output, the coal industry requires 30 cents of coal and 40 cents of steel.

To produce one dollar’s worth of output, the steel industry requires 20 cents of steel and 40 cents of coal.

(a) Find D, the input/output matrix for this system.

(b) Then solve for the (gross) output matrix X in the equation $X = D X + E$ where the external demand is given by $E = \begin{bmatrix} 40{,}000 \\ 50{,}000 \end{bmatrix}$