Math 202 | Problem Set for Chapter 3 | C.S.Davis

Name ____

Do as many of these problems as you can by hand in preparation for those on the hour quiz that you will need to do by hand. Then you can use the calculator to check your work if you wish.

1. Give a definition for the determinant of a matrix A. (If you use the letters C or M in your definition, define them as well.)

2. Use row operations to form a triangular matrix to find the determinant of the matrix below.

$$A = \begin{bmatrix} 0 & 4 & 12 \\ 2 & 4 & 6 \\ -4 & 1 & 6 \end{bmatrix}$$

3. Evaluate the following determinant of the following matrix by expanding by cofactors. Expand by the easiest row or column.

$$\begin{bmatrix} 3 & -1 & 0 & 5 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & -3 & 0 \end{bmatrix}$$

4. Suppose the row operations associated with these elementary matrices are used to row reduce a square matrix A into a triangular matrix B. The value of det(B) = 12. Find the value of det(A).

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

5. A matrix A is said to be an idempotent matrix if $A^2 = A$. If A is given to be idempotent, use determinants to prove that the determinant is either zero or one.

6. Consider the matrix $A = \begin{bmatrix} 3 & 10 & 80 & 44 \\ 0 & -2 & 36 & 16 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -2 \end{bmatrix}$

a. Use a single row operation on the determinant of A to make the evaluation of the determinant easier. Then give the value of the determinant of A.

b. Use the value of the determinant above to conclude whether the matrix A has an inverse, and if so, find the value of the determinant of the inverse of A.

7. For the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

a. Write the adjoint of A in determinant form.

b. Find A^{-1} using the adjoint of A.

8. Give the y value of the solution of this system by Cramer's Rule:

$$7x - 2y = 4$$
$$3x + 4y = 1$$

$y =$

9. Give the area of the triangle whose vertices are (2,1), (6,4) and (7,-1) using determinants.

10. Give the determinant form of the equation of the plane passing through the points: (1,2,-3), (4,-1,0), (2,0,1) and evaluate it to obtain the general form, $ax+by+cz=d$.

11. Consider the matrix $A = \begin{bmatrix} 3 & 3 \\ -2 & -4 \end{bmatrix}$

a. Find all real eigenvalues for the matrix A.

b. Find the eigenvectors of A corresponding to the eigenvalues.

c. Give the definition of eigenvalue and eigenvector and demonstrate that one of your eigenvalues and its eigenvector satisfies the definition for this matrix A.

12. Let A be an n x n matrix. Name the 5 conditions that are equivalent to "A is invertible. Use the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ and/or vector $\mathbf{b} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ to demonstrate the conditions.

a. A is invertible (or A is non-singular).

b.

c.

d.

e.

f.