Name ________________________________

1. For the vectors **u** = (1,4,2) and **v** = (3,-2,0),

   a. give 4**u** -5**v**;

   b. determine whether the vector **w** = (5,6,4) is a linear combination of **u** and **v**, and if so, write **w** as a linear combination of **u** and **v**.

   c. repeat part (b) with the vector **x** = (1,2,3).

2. Complete the proof of the cancellation property of vector addition by supplying the justification for each step. Prove that if **u, v**, and **w** are vectors in a vector space V such that **u** + **w** = **v** + **w**, then u = v.

u + **w** = **v** + **w**	Given
u + **w** + (-**w**) = **v** + **w** + (-**w**)	________________
u + (**w** +(-**w**)) = **v** + (**w** +(-**w**))	________________
u + **0** = **v** + **0**	________________
u= **v**	________________

2. State two theorems concerning what it takes for a subset of a vector space to be a subspace. Apply the theorems to determine whether the following subsets of vector spaces is a vector subspace.

a. the set of tenth degree polynomials is a vector subspace of the vector space of all polynomials under the usual operations. Give reasons for your decision.

b. the intersection of the two planes given by the two equations $x+y+z = 0$ and $2x+3y+5z=0$

c. the plane given by the equation $6x-3y+2z=18$.

d. the set of 2-by-2 matrices with a zero in the left lower corner is a subspace of the vector space of all 2-by-2 matrices under the usual operations.

3. For the vectors (1,2,3), (4,5,6) and (7,8,9), do the following. Do work or give reasons to support your conclusion.

   a. Determine whether these vectors are linearly independent or linearly dependent. If linearly dependent, find a linear combination that demonstrates that these vectors are linearly dependent.

   b. Determine whether the vectors span $R^{3.}$

   c. Determine whether the vectors are a basis for R^3.

   d. Give the dimension of the vector subspace spanned by the three vectors.

   e. If the three vectors are not a basis, find a basis for the subspace spanned by the three vectors.

4. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 4 & 2 & 1 \\ 4 & 3 & 9 & 4 & 4 \\ 3 & 1 & 5 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

a. Find a basis for the row space of A.

b. Find a basis for the column space of A two ways, with and without using the transpose.

c. Give a basis for the nullspace of A. (This is also called the solution space for A$\mathbf{x}$ = $\mathbf{0}$.)

d. Give the rank of A, the nullity of A and the relationship between them and the number n of columns of A.

5. Consider the underlying vector space $V=R^3$ with the standard basis S = { (1,0,0), (0,1,0),(0,0,1) }
Let B = { (1,2,3), (1,0,1) } be a basis for a vector subspace W of $V=R^3$.
Let D = { (4,2,6), (7,4,11) } be another basis for the vector subspace W of V.
Let $[\mathbf{x}]_B = (6,-2)$ or $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ be the coordinate vector of **x** relative to the basis B.

Show the work that supports your answer each of the following.

a. Find the transition matrix P^{-1} from B to D and the transition matrix P from D to B.

b. Show that the two transition matrices work using Bcol, Dcol, P and P^{-1}.

c. Tell what it means to say that $[\mathbf{x}]_B = (6,-2)$ is the coordinate vector of **x** relative to the basis B.

d. Find $[\mathbf{x}]_S$, the coordinate vector of **x** relative to the standard basis S.

e. Use P or P^{-1} as appropriate to find, $[\mathbf{x}]_D$, the coordinate vector of **x** relative to the basis D, from $[\mathbf{x}]_B = (6,-2)$.

6. Prove that M_{2x2} with the standard operations of addition and scalar multiplication satisfies the associative property of addition. Give reasons for every step. (Hint: The beginning of your proof should look like this.

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, B = ..., C = ...

$A + (B + C) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} +(\ldots\ldots\ldots)$

= ....